3. Bez Bruchgleichungen Lösungen AB 3

1.

$$\begin{aligned} \frac{1}{3}-\frac{4}{x} &= 1 && /\cdot 3x \\ x-12 &= 3x && /-x \\ -12 &= 2x && /:2 \\ -6 &= x \end{aligned}$$

$x \neq 0$

$\underline{\underline{L=\{-6\}}}$

2.

$$\begin{aligned} \frac{x^2-4}{3x}-\frac{2-x}{3} &= \frac{2x}{3}-\frac{4}{x} && /\cdot 3x \\ x^2-4-\mathrm{x}(2-\mathrm{x}) &= 2x^2-12 \\ x^2-4-2\mathrm{x}+\mathrm{x}^2 &= 2x^2-12 \\ 2x^2-2x-4 &= 2x^2-12 && /-2\mathrm{x}^2 \\ -2x-4 &= -12 && /+2\mathrm{x} \\ -4 &= 2x-12 && /+12 \\ 8 &= 2x && /:2 \\ 4 &= x \end{aligned}$$

$x \neq 0$

$\underline{\underline{L=\{4\}}}$

3.

$$\begin{aligned} \frac{17}{y}-\left(\frac{9}{2y}-5\right) &= 5 && /\cdot 2y \\ 34-(9-10\mathrm{y}) &= 10y \\ 34-9+10\mathrm{y} &= 10y \\ 25+10\mathrm{y} &= 10y && /-10y \\ 25 &= 0 \end{aligned}$$

$y \neq 0$

$\underline{\underline{L=\{\,\}}}$

4.

$$\begin{aligned} \frac{4}{x+5} &= 1 && /\cdot (x+5) \\ 4 &= x+5 && /-5 \\ -1 &= x \end{aligned}$$

$x \neq -5$

$\underline{\underline{L=\{-1\}}}$

5. $\frac{4}{y} = \frac{10}{y+4}$ $/\cdot y(y+4)$ $\underline{y \neq 0}$ $\underline{y \neq -4}$

$4(y+4) = 10y$

$4y+16 = 10y$ $/-4y$

$16 = 6y$ $/:6$

$\frac{8}{3} = y$

$\underline{\underline{L = \left\{\frac{8}{3}\right\}}}$

6. $\frac{3}{5x+7} = \frac{1}{2x}$ $/\cdot 2x(5x+7)$ $\underline{x \neq 0}$ $\underline{x \neq -\frac{7}{5}}$

$6x = 5x+7$ $/-5x$

$x = 7$

$\underline{\underline{L = \{7\}}}$

7. $\frac{1}{x-2} = \frac{2}{x^2-2x}$

$\frac{1}{(x-2)} = \frac{2}{x(x-2)}$ $/\cdot x(x-2)$ $\underline{x \neq 0}$ $\underline{x \neq 2}$

$x = 2$

$\underline{\underline{L = \{\}}}$

8. $\frac{4}{y+1} = \frac{10}{y+4}$ $/\cdot (y+1)(y+4)$ $\underline{y \neq -1}$ $\underline{y \neq -4}$

$4(y+4) = 10(y+1)$

$4y+16 = 10y+10$ $/-4y$

$16 = 6y+10$ $/-10$

$6 = 6y$ $/:6$

$1 = y$

$\underline{\underline{L = \{1\}}}$

9. $\frac{1}{3-x} + \frac{1}{x+4} = \frac{7}{(3-x)(x+4)}$ $/\cdot (3-x)(x+4)$ $\underline{x \neq 3}$ $\underline{x \neq -4}$

$x+4+3-x = 7$

$7 = 7$

$\underline{\underline{L = \{\mathbb{Q} \,/\, 3; -4\}}}$ (alle Zahlen ausser 3 und -4)

10. $\frac{3}{(x-2)(x+3)} = \frac{4}{x+3} - \frac{5}{x-2}$ $/\cdot (x-2)(x+3)$ $\underline{x \neq 2}$ $\underline{x \neq -3}$

$$\begin{aligned} 3 &= 4(x-2) - 5(x+3) \\ 3 &= 4x - 8 - 5x - 15 \\ 3 &= -x - 23 \quad /+x \\ x+3 &= -23 \quad /-3 \\ x &= -26 \end{aligned}$$

$$\underline{\underline{L = \{-26\}}}$$

11. $\frac{1}{8} = \frac{3}{4x} + \frac{1}{2x}$ $/\cdot 8x$ $\underline{x \neq 0}$

$$\begin{aligned} x &= 6 + 4 \\ x &= 10 \end{aligned}$$

$$\underline{\underline{L = \{10\}}}$$

12. $\frac{1}{2x} = \frac{1}{3x}$ $/\cdot 6x$ $\underline{x \neq 0}$

$$3 = 2$$

$$\underline{\underline{L = \{\,\}}}$$

13. $\frac{4}{9x} - \frac{2}{15x} = \frac{1}{3}$ $/\cdot 45x$ $\underline{x \neq 0}$

$$\begin{aligned} 20 - 6 &= 15x \\ 14 &= 15x \quad /:15 \\ \frac{14}{15} &= x \end{aligned}$$

$$\underline{\underline{L = \left\{\frac{14}{15}\right\}}}$$

14. $\frac{5}{x} - \frac{4}{21} = 1 - \frac{10}{3x}$ $/\cdot 21x$ $\underline{x \neq 0}$

$$\begin{aligned} 105 - 4x &= 21x - 70 \quad /+4x \\ 105 &= 25x - 70 \quad /+70 \\ 175 &= 25x \quad /:25 \\ 7 &= x \end{aligned}$$

$$\underline{\underline{L = \{7\}}}$$

15.

$$\frac{1}{x}-\left(\frac{8}{9}-\frac{1}{3x}\right) = 0$$

$$\frac{1}{x}-\frac{8}{9}+\frac{1}{3x} = 0 \qquad /\cdot 9x \qquad \underline{x \neq 0}$$

$$9-8x+3 = 0$$

$$12-8x = 0 \qquad /+8x$$

$$12 = 8x \qquad /:8$$

$$1{,}5 = x$$

$$\underline{\underline{L=\{1{,}5\}}}$$

16.

$$\frac{6-5x}{3}-\left(\frac{1}{x}+\frac{1}{4x}\right) = -\frac{10x+3}{6}$$

$$\frac{6-5x}{3}-\frac{1}{x}-\frac{1}{4x} = -\frac{10x+3}{6} \qquad /\cdot 12x \qquad \underline{x \neq 0}$$

$$4x(6-5x)-12-3 = -2x(10x+3) \qquad /+x$$

$$24x-20x^2-15 = -20x^2-6x \qquad /+20x^2$$

$$24x-15 = -6x \qquad /+6x$$

$$30x-15 = 0 \qquad /+15$$

$$30x = 15 \qquad /:30$$

$$x = 0{,}5$$

$$\underline{\underline{L=\{0{,}5\}}}$$

17.

$$\frac{13(13-x)}{3x} = \frac{3}{4x}+\frac{21}{x}+\frac{31}{12} \qquad /\cdot 12x \qquad \underline{x \neq 0}$$

$$52(13-x) = 9+252+31x$$

$$676-52x = 261+31x \qquad /+52x$$

$$676 = 261+83x \qquad /-261$$

$$415 = 83x \qquad /:83$$

$$5 = x$$

$$\underline{\underline{L=\{5\}}}$$

18.

$$0 = \frac{3}{x} - \left(\frac{1}{3x} + \frac{6-5x}{2x} + \frac{2}{3}\right)$$
$$0 = \frac{3}{x} - \frac{1}{3x} - \frac{6-5x}{2x} - \frac{2}{3} \qquad /\cdot 6x \qquad \underline{x \neq 0}$$
$$0 = 18 - 2 - 3(6-5x) - 4x$$
$$0 = 18 - 2 - 18 + 15x - 4x$$
$$0 = 11x - 2 \qquad /+2$$
$$2 = 11x \qquad /:11$$
$$\frac{2}{11} = x$$

$$\underline{\underline{L = \left\{\frac{2}{11}\right\}}}$$

19.

$$17{,}5 = \frac{6}{v} - \frac{1{,}5}{v} + \frac{4}{3v} \qquad /\cdot 3v \qquad \underline{v \neq 0}$$
$$52{,}5v = 18 - 4{,}5 + 4$$
$$52{,}5v = 17{,}5 \qquad /:52{,}5$$
$$v = \frac{1}{3}$$

$$\underline{\underline{L = \left\{\frac{1}{3}\right\}}}$$

20.

$$\frac{7}{a-3} = 2 \qquad /\cdot (a-3) \qquad \underline{a \neq 3}$$
$$7 = 2(a-3)$$
$$7 = 2a - 6 \qquad /+6$$
$$13 = 2a \qquad /:2$$
$$6{,}5 = a$$

$$\underline{\underline{L = \{6{,}5\}}}$$

21.

$$\frac{2}{t+1} = \frac{1}{4} \qquad /\cdot 4(t+1) \qquad \underline{t \neq -1}$$
$$8 = t + 1 \qquad /-1$$
$$7 = t$$

$$\underline{\underline{L = \{7\}}}$$

22. $\frac{3}{u+2} + 2{,}5 = 0 \quad /\cdot (u+2) \quad \underline{u \neq -2}$

$3 + 2{,}5(u+2) = 0$

$3 + 2{,}5u + 5 = 0$

$2{,}5u + 8 = 0 \quad /-8$

$2{,}5u = -8 \quad /:2{,}5$

$u = -3{,}2$

$\underline{\underline{L = \{-3{,}2\}}}$

23. $\frac{2}{t} = \frac{1}{t-1} \quad /\cdot t(t-1) \quad \underline{t \neq 0} \quad \underline{t \neq 1}$

$2(t-1) = t$

$2t - 2 = t \quad /-t$

$t - 2 = 0 \quad /+2$

$t = 2$

$\underline{\underline{L = \{2\}}}$

24. $\frac{1}{x} - \frac{1}{x+1} = 0 \quad /\cdot x(x+1) \quad \underline{x \neq 0} \quad \underline{x \neq -1}$

$x + 1 - x = 0$

$1 = 0$

$\underline{\underline{L = \{\,\}}}$

25. $\frac{5}{x+8} = \frac{1}{x} \quad /\cdot x(x+8) \quad \underline{x \neq 0} \quad \underline{x \neq -8}$

$5x = x + 8 \quad /-x$

$4x = 8 \quad /:4$

$x = 2$

$\underline{\underline{L = \{2\}}}$

26. $\frac{5}{48-x} = \frac{3}{x} \quad /\cdot x(48-x) \quad \underline{x \neq 48} \quad \underline{x \neq 0}$

$5x = 3(48-x)$

$5x = 144 - 3x \quad /+3x$

$8x = 144 \quad /:8$

$x = 18$

$\underline{\underline{L = \{18\}}}$

27. $\frac{5}{x}-\frac{4}{x+1}=\frac{4}{x(x+1)}$ $/\cdot x(x+1)$ $\underline{x\neq 0}$ $\underline{x\neq -1}$

$$\begin{aligned} 5(x+1)-4x &= 4 \\ 5x+5-4x &= 4 \\ x+5 &= 4 \quad /-5 \\ x &= -1 \end{aligned}$$

$$\underline{\underline{L=\{\,\}}}$$

28. $\frac{3}{x-2}+\frac{4}{x}=\frac{6}{x(x-2)}$ $/\cdot x(x-2)$ $\underline{x\neq 0}$ $\underline{x\neq 2}$

$$\begin{aligned} 3x+4(x-2) &= 6 \\ 3x+4x-8 &= 6 \\ 7x-8 &= 6 \quad /+8 \\ 7x &= 14 \quad /:7 \\ x &= 2 \end{aligned}$$

$$\underline{\underline{L=\{\,\}}}$$

29. $\frac{5}{4t+t^2}-\frac{1}{t}=0$

$\frac{5}{t(4+t)}-\frac{1}{t}=0$ $/\cdot t(4+t)$ $\underline{t\neq 0}$ $\underline{t\neq -4}$

$$\begin{aligned} 5-(4+t) &= 0 \\ 5-4-t &= 0 \quad /+t \\ 1 &= t \end{aligned}$$

$$\underline{\underline{L=\{1\}}}$$

30. $\frac{0{,}5}{y-1}=\frac{1}{2y-2}$

$\frac{0{,}5}{(y-1)}=\frac{1}{2(y-1)}$ $/\cdot 2(y-1)$ $\underline{y\neq 1}$

$1=1$

$\underline{\underline{L=\{\mathbb{Q}\,/\,1\}}}$ (alle Zahlen ausser 1)