3. Bez Doppelbruchgleichungen Lösungen AB 5

1.
$$\frac{\frac{x}{a}+\frac{a}{b}}{\frac{x}{b}-\frac{b}{a}} - \frac{a}{a-b} = 0$$
$$\frac{\frac{bx+a^2}{ab}}{\frac{ax-b^2}{ab}} - \frac{a}{a-b} = 0$$
$$\frac{(bx+a^2)ab}{ab(ax-b^2)} - \frac{a}{a-b} = 0$$
$$\frac{(bx+a^2)}{(ax-b^2)} - \frac{a}{a-b} = 0 \qquad /\cdot (ax-b^2)(a-b)$$
$$(bx+a^2)(a-b) - a(ax-b^2) = 0$$
$$abx - b^2x + a^3 - a^2b - a^2x + ab^2 = 0 \qquad /+a^2x - abx + b^2x$$
$$a^3 - a^2b + ab^2 = a^2x - abx + b^2x$$
$$a(a^2 - ab + b^2) = x(a^2 - ab + b^2) \qquad /:(a^2 - ab + b^2)$$
$$\underline{\underline{a = x}}$$

2.
$$\frac{2b}{ax-ab} - \frac{4a}{bx+b^2} = \frac{bx-4a^2+4b^2}{ax^2-ab^2}$$
$$\frac{2b}{a(x-b)} - \frac{4a}{b(x+b)} = \frac{bx-4a^2+4b^2}{a(x^2-b^2)}$$
$$\frac{2b}{a(x-b)} - \frac{4a}{b(x+b)} = \frac{bx-4a^2+4b^2}{a(x+b)(x-b)} \qquad /\cdot ab(x-b)(x+b)$$
$$2b^2(x+b) - 4a^2(x-b) = b(bx - 4a^2 + 4b^2)$$
$$2b^2x + 2b^3 - 4a^2x + 4a^2b = b^2x - 4a^2b + 4b^3 \qquad /-2b^3 - 4a^2b - b^2x$$
$$b^2x - 4a^2x = 2b^3 - 8a^2b$$
$$x(b^2 - 4a^2) = 2b(b^2 - 4a^2) \qquad /:(b^2 - 4a^2)$$
$$\underline{\underline{x = 2b}}$$

3.
$$\frac{a^2b-x}{a} + \frac{b^2c-x}{b} + \frac{ac^2-x}{c} = 0 \qquad /\cdot abc$$
$$bc(a^2b - x) + ac(b^2c - x) + ab(ac^2 - x) = 0$$
$$a^2b^2c - bcx + ab^2c^2 - acx + a^2bc^2 - abx = 0 \qquad /+abx + acx + bcx$$
$$a^2b^2c + ab^2c^2 + a^2bc^2 = abx + bcx + acx$$
$$abc(ab + bc + ac) = x(ab + bc + ac) \qquad /:(ab + bc + ac)$$
$$\underline{\underline{abc = x}}$$

4.

$$\frac{\frac{x}{a}+\frac{1}{b}}{\frac{x}{b}+\frac{1}{a}} = \frac{a-2b}{b-2a}$$

$$\frac{\frac{bx+a}{ab}}{\frac{ax+b}{ab}} = \frac{(a-2b)}{(b-2a)}$$

$$\frac{(bx+a)\cdot ab}{ab(ax+b)} = \frac{(a-2b)}{(b-2a)}$$

$$\frac{(bx+a)}{(ax+b)} = \frac{(a-2b)}{(b-2a)} \qquad /\cdot (ax+b)(b-2a)$$

$$(bx+a)(b-2a) = (a-2b)(ax+b)$$

$$b^2x - 2abx + ab - 2a^2 = a^2x + ab - 2abx - 2b^2 \qquad /-ab + 2abx$$

$$b^2x - 2a^2 = a^2x - 2b^2 \qquad /+2b^2 - b^2x$$

$$-2a^2 + 2b^2 = a^2x - b^2x$$

$$-2(a^2 - b^2) = x(a^2 - b^2) \qquad /:(a^2 - b^2)$$

$$\underline{\underline{-2 = x}}$$

5.

$$\frac{\frac{x-a}{x+2a}+8}{2+\frac{x+2a}{x-3a}} = 3$$

$$\frac{\frac{x-a+8(x+2a)}{(x+2a)}}{\frac{2(x-3a)+x+2a}{(x-3a)}} = 3$$

$$\frac{\frac{x-a+8x+16a}{(x+2a)}}{\frac{2x-6a+x+2a}{(x-3a)}} = 3$$

$$\frac{\frac{9x+15a}{(x+2a)}}{\frac{3x-4a}{(x-3a)}} = 3$$

$$\frac{\frac{3(3x+5a)}{(x+2a)}}{\frac{(3x-4a)}{(x-3a)}} = 3$$

$$\frac{3(3x+5a)(x-3a)}{(x+2a)(3x-4a)} = 3 \qquad /\cdot (x+2a)(3x-4a)$$

$$3(3x+5a)(x-3a) = 3(x+2a)(3x-4a) \qquad /:3$$

$$3x^2 - 9ax + 5ax - 15a^2 = 3x^2 - 4ax + 6ax - 8a^2 \qquad /-3x^2 + 8a^2$$

$$-4ax - 7a^2 = 2ax \qquad /+4ax$$

$$-7a^2 = 6ax \qquad /:6a$$

$$\underline{\underline{-\frac{7a}{6} = x}}$$

6.

$$\frac{2a-x}{bx-ab}-\frac{b}{ax-a^2}-\frac{a+b-x}{bx} = 0$$

$$\frac{2a-x}{b(x-a)}-\frac{b}{a(x-a)}-\frac{a+b-x}{bx} = 0 \qquad /\cdot abx(x-a)$$

$$ax(2a-x)-b^2x-a(a+b-x)(x-a) = 0$$

$$2a^2x-ax^2-b^2x-a(ax-a^2+bx-ab-x^2+ax) = 0$$

$$2a^2x-ax^2-b^2x-a^2x+a^3-abx+a^2b+ax^2-a^2x = 0$$

$$-b^2x+a^3-abx+a^2b = 0 \qquad /+abx+b^2x$$

$$+a^3+a^2b = abx+b^2x$$

$$a^2(a+b) = bx(a+b) \qquad /:b(a+b)$$

$$\frac{a^2(a+b)}{b(a+b)} = x$$

$$\underline{\underline{\frac{a^2}{b} = x}}$$

7.

$$\frac{a+4x}{a-2x}-\frac{a+2x}{a-x} = \frac{a^2+x^2}{a^2-3ax+2x^2}-\frac{1}{2}$$

$$\frac{a+4x}{(a-2x)}-\frac{a+2x}{(a-x)} = \frac{a^2+x^2}{(a-2x)(a-x)}-\frac{1}{2} \qquad /\cdot 2(a-2x)(a-x)$$

$$2(a+4x)(a-x)-2(a+2x)(a-2x) = 2(a^2+x^2)-(a-2x)(a-x)$$

$$2(a^2-ax+4ax-4x^2)-2(a^2-4x^2) = 2a^2+2x^2-(a^2-3ax+2x^2)$$

$$2(a^2+3ax-4x^2)-2a^2+8x^2 = 2a^2+2x^2-a^2+3ax-2x^2$$

$$2a^2+6ax-8x^2-2a^2+8x^2 = a^2+3ax$$

$$6ax = a^2+3ax \qquad /-3ax$$

$$3ax = a^2 \qquad /:3a$$

$$\underline{\underline{x = \frac{a}{3}}}$$

8.

$$\frac{a}{b(a-x)}+\frac{c}{d(x-a)} = \frac{ad-bc}{3abd}$$

$$\frac{a}{b(a-x)}-\frac{c}{d(a-x)} = \frac{ad-bc}{3abd} \qquad /\cdot 3abd(a-x)$$

$$3a^2d-3abc = (ad-bc)(a-x)$$

$$3a^2d-3abc = a^2d-adx-abc+bcx \qquad /-a^2d+abc$$

$$2a^2d-2abc = bcx-adx$$

$$-2a(bc-ad) = x(bc-ad) \qquad /:(bc-ad)$$

$$\underline{\underline{-2a = x}}$$

9.

$$\frac{x+1}{b-2}-\frac{2x-1}{3b+6} = \frac{2x}{b^2-4}+\frac{x+2}{2b-4}$$

$$\frac{x+1}{(b-2)}-\frac{2x-1}{3(b+2)} = \frac{2x}{(b+2)(b-2)}+\frac{x+2}{2(b-2)} \qquad /\cdot 6(b+2)(b-2)$$

$$6(b+2)(x+1)-2(b-2)(2x-1) = 12x+3(b+2)(x+2)$$

$$6(bx+b+2x+2)-2(2bx-b-4x+2) = 12x+3(bx+2b+2x+4)$$

$$6bx+6b+12x+12-4bx+2b+8x-4 = 12x+3bx+6b+6x+12$$

$$2bx+8b+20x+8 = 18x+3bx+6b+12 \qquad /-18x-3bx$$

$$-bx+8b+2x+8 = 6b+12 \qquad /-8b-8$$

$$-bx+2x = -2b+4$$

$$x(-b+2) = 2(-b+2) \qquad /:(-b+2)$$

$$\underline{\underline{x = 2}}$$