

3. Bez

Bruchgleichungen

Lösungen AB 4

$$\begin{aligned} 1. \quad \frac{x+3}{x-5} &= \frac{x-2}{x-6} && / \cdot (x-5)(x-6) && \underline{x \neq 5} \wedge \underline{x \neq 6} \\ (x+3)(x-6) &= (x-2)(x-5) \\ x^2 - 6x + 3x - 18 &= x^2 - 5x - 2x + 10 && / -x^2 \\ -3x - 18 &= -7x + 10 && / +7x \\ 4x - 18 &= 10 && / +18 \\ 4x &= 28 && / :4 \\ \underline{x} &= \underline{7} \end{aligned}$$

$$\begin{aligned} 2. \quad \frac{x-5}{x-3} &= \frac{1}{2x-2} + \frac{x-7}{x-1} && / \cdot 2(x-3)(x-1) \\ \frac{x-5}{(x-3)} &= \frac{1}{2(x-1)} + \frac{x-7}{(x-1)} && \underline{x \neq 3} \wedge \underline{x \neq 1} \\ 2(x-1)(x-5) &= x-3 + 2(x-7)(x-3) \\ 2(x^2 - 5x - x + 5) &= x-3 + 2(x^2 - 3x - 7x + 21) \\ 2(x^2 - 6x + 5) &= x-3 + 2(x^2 - 10x + 21) \\ 2x^2 - 12x + 10 &= x-3 + 2x^2 - 20x + 42 && / -2x^2 \\ -12x + 10 &= -19x + 39 && / +19x \\ 7x + 10 &= 39 && / -10 \\ 7x &= 29 && / :7 \\ x &= \frac{29}{7} \end{aligned}$$

$$\begin{aligned} 3. \quad \frac{1}{x-1} + \frac{2}{x+1} - \frac{11}{x^2-1} &= 0 \\ \frac{1}{x-1} + \frac{2}{x+1} - \frac{11}{(x-1)(x+1)} &= 0 && / \cdot (x+1)(x-1) && \underline{x \neq 1} \wedge \underline{x \neq -1} \\ x+1 + 2(x-1) - 11 &= 0 \\ x+1 + 2x - 2 - 11 &= 0 \\ 3x - 12 &= 0 && / +12 \\ 3x &= 12 && / :3 \\ \underline{x} &= \underline{4} \end{aligned}$$

$$\begin{aligned} 4. \quad \frac{3}{z+4} - \frac{2}{z-4} &= \frac{5z-20}{z^2-16} \\ \frac{3}{z+4} - \frac{2}{z-4} &= \frac{5z-20}{(z+4)(z-4)} && / \cdot (z+4)(z-4) && \underline{x \neq 4} \wedge \underline{x \neq -4} \\ 3(z-4) - 2(z+4) &= 5z-20 \\ 3z - 12 - 2z - 8 &= 5z - 20 \\ z - 20 &= 5z - 20 && / -z \\ -20 &= 4z - 20 && / +20 \\ 0 &= 4z && / :4 \\ \underline{0} &= \underline{z} \end{aligned}$$

$$\begin{aligned}
5. \quad \frac{2+7x}{1+x} &= \frac{4-9x}{1-x} - \frac{12-2x^2}{1-x^2} \\
\frac{2+7x}{(1+x)} &= \frac{4-9x}{(1-x)} - \frac{12-2x^2}{(1-x)(1+x)} && / \cdot (1+x)(1-x) \quad \underline{x \neq 1} \wedge \underline{x \neq -1} \\
(2+7x)(1-x) &= (4-9x)(1+x) - (12-2x^2) \\
2-2x+7x-7x^2 &= 4+4x-9x-9x^2-12+2x^2 \\
2+5x-7x^2 &= -7x^2-5x-8 && /+7x^2 \\
2+5x &= -5x-8 && /+5x \\
2+10x &= -8 && /-2 \\
10x &= -10 && /:10 \\
x &= -1 \\
\mathbf{\underline{\underline{L = \{ \} }}}
\end{aligned}$$

$$\begin{aligned}
6. \quad \frac{1}{x-3} - \frac{2}{x+3} &= \frac{3}{x^2-9} \\
\frac{1}{(x-3)} - \frac{2}{(x+3)} &= \frac{3}{(x+3)(x-3)} && / \cdot (x+3)(x-3) \quad \underline{x \neq 3} \wedge \underline{x \neq -3} \\
x+3-2(x-3) &= 3 \\
x+3-2x+6 &= 3 \\
-x+9 &= 3 && /+x \\
9 &= x+3 && /-3 \\
\mathbf{\underline{\underline{6 = x}}}
\end{aligned}$$

$$\begin{aligned}
7. \quad \frac{x}{x-1} + \frac{x+1}{3(x-1)} - \frac{x^2}{x^2-2x+1} &= \frac{1}{3} \\
\frac{x}{(x-1)} + \frac{x+1}{3(x-1)} - \frac{x^2}{(x-1)(x-1)} &= \frac{1}{3} && / \cdot 3(x-1)(x-1) \quad \underline{x \neq 1} \\
3x(x-1) + (x+1)(x-1) - 3x^2 &= (x-1)(x-1) \\
3x^2 - 3x + x^2 - 1 - 3x^2 &= x^2 - 2x + 1 && /-x^2 \\
-3x - 1 &= -2x + 1 && /+3x \\
-1 &= x + 1 && /-1 \\
\mathbf{\underline{\underline{-2 = x}}}
\end{aligned}$$

$$\begin{aligned}
8. \quad \frac{2}{x} &= \frac{1}{(x-1)} + \frac{1}{(x+2)} - \frac{6}{x(x-1)(x+2)} && / \cdot x(x-1)(x+2) \quad \underline{x \neq 0} \\
2(x-1)(x+2) &= x(x+2) + x(x-1) - 6 && \underline{x \neq 1} \wedge \underline{x \neq -2} \\
2(x^2+2x-x-2) &= x^2+2x+x^2-x-6 \\
2(x^2+x-2) &= 2x^2+x-6 \\
2x^2+2x-4 &= 2x^2+x-6 && /-2x^2 \\
2x-4 &= x-6 && /-x \\
x-4 &= -6 && /+4 \\
x &= -2 \\
\mathbf{\underline{\underline{L = \{ \} }}}
\end{aligned}$$

$$\begin{aligned}
9. \quad & \frac{1}{a-2} + \frac{1}{a+1} = \frac{2}{a-1} && / \cdot (a-2)(a-1)(a+1) && \underline{a \neq 2} \\
& (a-1)(a+1) + (a-2)(a-1) = 2(a-2)(a+1) && && \underline{a \neq 1} \wedge \underline{a \neq -1} \\
& a^2 - 1 + a^2 - a - 2a + 2 = 2(a^2 + a - 2a - 2) \\
& 2a^2 - 3a + 1 = 2(a^2 - a - 2) \\
& 2a^2 - 3a + 1 = 2a^2 - 2a - 4 && / -2a^2 \\
& -3a + 1 = -2a - 4 && / +3a \\
& 1 = a - 4 && / +4 \\
& \underline{\underline{5 = a}}
\end{aligned}$$

$$\begin{aligned}
10. \quad & \frac{3}{p+2} + \frac{4}{p-1} - \frac{7}{p-2} = 0 && / \cdot (p-2)(p+2)(p-1) \\
& 3(p-2)(p-1) + 4(p-2)(p+2) - 7(p+2)(p-1) = 0 && \underline{p \neq 2} \\
& 3(p^2 - p - 2p + 2) + 4(p^2 - 4) - 7(p^2 - p + 2p - 2) = 0 && \underline{p \neq -2} \\
& 3(p^2 - 3p + 2) + 4(p^2 - 4) - 7(p^2 + p - 2) = 0 && \underline{p \neq 1} \\
& 3p^2 - 9p + 6 + 4p^2 - 16 - 7p^2 - 7p + 14 = 0 \\
& -16p + 4 = 0 && / +16p \\
& 4 = 16p && / :16 \\
& \underline{\underline{0,25 = p}}
\end{aligned}$$

$$\begin{aligned}
11. \quad & \frac{4}{x+3} + \frac{3}{2x} - \frac{31}{2x(x+3)} = 0 && / \cdot 2x(x+3) && \underline{x \neq 0} \wedge \underline{x \neq -3} \\
& 8x + 3(x+3) - 31 = 0 \\
& 8x + 3x + 9 - 31 = 0 \\
& 11x - 22 = 0 && / +22 \\
& 11x = 22 && / :11 \\
& \underline{\underline{x = 2}}
\end{aligned}$$

$$\begin{aligned}
12. \quad & \frac{1}{x-1} - \frac{2}{x^2} = \frac{1}{x} && / \cdot x^2(x-1) && \underline{x \neq 0} \wedge \underline{x \neq 1} \\
& x^2 - 2(x-1) = x(x-1) \\
& x^2 - 2x + 2 = x^2 - x && / -x^2 \\
& -2x + 2 = -x && / +2x \\
& \underline{\underline{2 = x}}
\end{aligned}$$

$$\begin{aligned}
13. \quad & \frac{5}{z-2} = \frac{6}{z-5} && / \cdot (z-2)(z-5) && \underline{z \neq 2} \wedge \underline{z \neq 5} \\
& 5(z-5) = 6(z-2) \\
& 5z - 25 = 6z - 12 && / -5z \\
& -25 = z - 12 && / +12 \\
& \underline{\underline{-13 = z}}
\end{aligned}$$

$$\begin{aligned}
14. \quad & \frac{3}{v-1} + \frac{4}{1-v} = 0 \\
& \frac{3}{(v-1)} - \frac{4}{(v-1)} = 0 & / \cdot (v-1) & \quad \underline{v \neq 1} \\
& 3 - 4 = 0 \\
& -1 = 0 & / 0 \\
& \underline{\underline{L = \{ \}}}
\end{aligned}$$

$$\begin{aligned}
15. \quad & \frac{5}{1-k} = \frac{-5}{k-1} \\
& \frac{5}{(1-k)} = \frac{5}{(1-k)} & / \cdot (1-k) & \quad \underline{k \neq 1} \\
& 5 = 5 \\
& \underline{\underline{L = \{\mathbb{Q} \setminus 1\}}}
\end{aligned}$$

$$\begin{aligned}
16. \quad & \frac{2x+3}{x-5} = \frac{6x-13}{3x+11} & / \cdot (x-5)(3x+11) & \quad \underline{x \neq 5} \wedge \underline{x \neq -\frac{11}{3}} \\
& (2x+3)(3x+11) = (6x-13)(x-5) \\
& 6x^2 + 22x + 9x + 33 = 6x^2 - 30x - 13x + 65 & / -6x^2 \\
& 31x + 33 = -43x + 65 & / +43x \\
& 74x + 33 = 65 & / -33 \\
& 74x = 32 & / : 74 \\
& x = \frac{16}{37} \\
& \underline{\underline{x}}
\end{aligned}$$

$$\begin{aligned}
17. \quad & \frac{2x-3}{x+3} = \frac{6x-13}{3x+11} & / \cdot (x+3)(3x+11) & \quad \underline{x \neq -3} \wedge \underline{x \neq -\frac{11}{3}} \\
& (2x-3)(3x+11) = (6x-13)(x+3) \\
& 6x^2 + 22x - 9x - 33 = 6x^2 + 18x - 13x - 39 & / -6x^2 \\
& 13x - 33 = 5x - 39 & / -5x \\
& 8x - 33 = -39 & / +33 \\
& 8x = -6 & / : 8 \\
& x = \frac{-3}{4} \\
& \underline{\underline{x}}
\end{aligned}$$

$$\begin{aligned}
18. \quad & \frac{9x-2}{12x-17} - \frac{6x+7}{8x-5} = 0 & / \cdot (12x-17)(8x-5) \\
& (9x-2)(8x-5) - (6x+7)(12x-17) = 0 & \quad \underline{x \neq \frac{17}{12}} \wedge \underline{x \neq \frac{5}{8}} \\
& 72x^2 - 45x - 16x + 10 - (72x^2 - 102x + 84x - 119) = 0 \\
& 72x^2 - 45x - 16x + 10 - 72x^2 + 102x - 84x + 119 = 0 \\
& -43x + 129 = 0 & / +43x \\
& 129 = 43x & / : 43 \\
& \underline{\underline{3 = x}}
\end{aligned}$$

$$\begin{aligned}
19. \quad & \frac{x+8}{3x+3} + \frac{x+2}{2x+2} = 1 \\
& \frac{x+8}{3(x+1)} + \frac{x+2}{2(x+1)} = 1 & / \cdot 6(x+1) & \quad \underline{x \neq -1} \\
& 2(x+8) + 3(x+2) = 6(x+1) \\
& 2x + 16 + 3x + 6 = 6x + 6 \\
& 5x + 22 = 6x + 6 & / -5x \\
& 22 = x + 6 & / -6 \\
& \underline{\underline{16 = x}}
\end{aligned}$$

$$\begin{aligned}
20. \quad & \frac{2x-3,5}{3-x} = -\frac{5}{2x-6} \\
& \frac{2x-3,5}{(3-x)} = -\frac{5}{2(x-3)} \\
& \frac{2x-3,5}{(3-x)} = \frac{5}{2(3-x)} & / \cdot 2(3-x) & \quad \underline{x \neq 3} \\
& 2(2x - 3,5) = 5 \\
& 4x - 7 = 5 & / +7 \\
& 4x = 12 & / :4 \\
& x = 3 \\
& \underline{\underline{L = \{ \}}}
\end{aligned}$$

$$\begin{aligned}
21. \quad & \frac{4+x}{x} = \frac{3+x}{x-1} & / \cdot x(x-1) & \quad \underline{x \neq 0} \wedge \underline{x \neq 1} \\
& (4+x)(x-1) = x(3+x) \\
& 4x - 4 + x^2 - x = 3x + x^2 & / -x^2 \\
& 3x - 4 = 3x & / -3x \\
& -4 = 0 \\
& \underline{\underline{L = \{ \}}}
\end{aligned}$$

$$\begin{aligned}
22. \quad & \frac{x}{x-1} = -\frac{1-x}{x} & / \cdot x(x-1) & \quad \underline{x \neq 0} \wedge \underline{x \neq 1} \\
& x^2 = -(1-x)(x-1) \\
& x^2 = (x-1)(x-1) \\
& x^2 = x^2 - 2x + 1 & / -x^2 \\
& 0 = -2x + 1 & / +2x \\
& 2x = 1 & / :2 \\
& x = \frac{1}{2} \\
& \underline{\underline{2}}
\end{aligned}$$

$$\begin{aligned}
23. \quad & \frac{x}{x+7} + \frac{x-4}{x-5} = 2 && / \cdot (x+7)(x-5) && \underline{x \neq -7 \wedge x \neq 5} \\
& x(x-5) + (x-4)(x+7) = 2(x+7)(x-5) \\
& x^2 - 5x + x^2 + 7x - 4x - 28 = 2(x^2 - 5x + 7x - 35) \\
& 2x^2 - 2x - 28 = 2x^2 - 10x + 14x - 70 && / -2x^2 \\
& -2x - 28 = 4x - 70 && / +2x \\
& -28 = 6x - 70 && / +70 \\
& 42 = 6x && / :6 \\
& \underline{\underline{7 = x}}
\end{aligned}$$

$$\begin{aligned}
24. \quad & \frac{7}{x-2} - \frac{5}{x+2} = \frac{30}{x^2-4} && / \cdot (x+2)(x-2) && \underline{x \neq 2 \wedge x \neq -2} \\
& \frac{7}{(x-2)} - \frac{5}{(x+2)} = \frac{30}{(x+2)(x-2)} \\
& 7(x+2) - 5(x-2) = 30 \\
& 7x + 14 - 5x + 10 = 30 && / 0 \\
& 2x + 24 = 30 && / -24 \\
& 2x = 6 && / :2 \\
& \underline{\underline{x = 3}}
\end{aligned}$$

$$\begin{aligned}
25. \quad & \frac{2}{2+y} - \frac{1}{y-2} = \frac{-4}{4-y^2} && / \cdot (2+y)(2-y) && \underline{y \neq 2 \wedge y \neq -2} \\
& \frac{2}{(2+y)} + \frac{1}{(2-y)} = \frac{-4}{(2+y)(2-y)} \\
& 2(2-y) + 2 + y = -4 \\
& 4 - 2y + 2 + y = -4 \\
& 6 - y = -4 && / +y \\
& 6 = y - 4 && / +4 \\
& \underline{\underline{10 = y}}
\end{aligned}$$

$$\begin{aligned}
26. \quad & \frac{3}{3x+4} - \frac{14}{3x-4} + \frac{35}{9x^2-16} = 0 && / \cdot (3x+4)(3x-4) && \underline{x \neq \frac{4}{3} \wedge x \neq -\frac{4}{3}} \\
& \frac{3}{(3x+4)} - \frac{14}{(3x-4)} + \frac{35}{(3x+4)(3x-4)} = 0 \\
& 3(3x-4) - 14(3x+4) + 35 = 0 \\
& 9x - 12 - 42x - 56 + 35 = 0 \\
& -33x - 33 = 0 && / +33x \\
& -33 = 33x && / :33 \\
& \underline{\underline{-1 = x}}
\end{aligned}$$

$$\begin{aligned}
27. \quad & \frac{39}{16x^2-25} - \frac{15}{4x-5} + \frac{52}{4x+5} = 0 \\
& \frac{39}{(4x+5)(4x-5)} - \frac{15}{(4x-5)} + \frac{52}{(4x+5)} = 0 \quad / \cdot (4x+5)(4x-5) \quad \underline{x \neq \frac{5}{4}} \wedge \underline{x \neq -\frac{5}{4}} \\
& 39 - 15(4x+5) + 52(4x-5) = 0 \\
& 39 - 60x - 75 + 208x - 260 = 0 \\
& 148x - 296 = 0 \quad /+296 \\
& 148x = 296 \quad /:148 \\
& \underline{\underline{x = 2}}
\end{aligned}$$

$$\begin{aligned}
28. \quad & \frac{(x-1)(x-2)}{x-3} = \frac{(x-3)^2}{x-6} \quad / \cdot (x-3)(x-6) \\
& (x-1)(x-2)(x-6) = (x-3)(x-3)(x-3) \quad \underline{x \neq 3} \wedge \underline{x \neq 6} \\
& (x^2 - 2x - x + 2)(x-6) = (x^2 - 3x - 3x + 9)(x-3) \\
& (x^2 - 3x + 2)(x-6) = (x^2 - 6x + 9)(x-3) \\
& x^3 - 3x^2 + 2x - 6x^2 + 18x - 12 = x^3 - 6x^2 + 9x - 3x^2 + 18x - 27 \quad /-x^3 \\
& -9x^2 + 20x - 12 = -9x^2 + 27x - 27 \quad /+9x^2 \\
& 20x - 12 = 27x - 27 \quad /-20x \\
& -12 = 7x - 27 \quad /+27 \\
& 15 = 7x \quad /:7 \\
& \underline{\underline{\frac{15}{7} = x}}
\end{aligned}$$