1. $$\frac{1-\frac{x+a}{x-b}}{1-\frac{x-a}{x+b}}+\frac{3bx-x^2}{bx-b^2}=$$

$$1-\frac{x+a}{x-b}=\frac{(x-b)}{(x-b)}-\frac{(x+a)}{(x-b)}=\frac{(x-b)-(x+a)}{(x-b)}=\frac{x-b-x-a}{(x-b)}=\frac{-b-a}{(x-b)}=\frac{-(a+b)}{(x-b)}$$

$$1-\frac{x-a}{x+b}=\frac{(x+b)}{(x+b)}-\frac{(x-a)}{(x+b)}=\frac{(x+b)-(x-a)}{(x+b)}=\frac{x+b-x+a}{(x+b)}=\frac{b+a}{(x+b)}=\frac{(a+b)}{(x+b)}$$

$$\frac{\frac{-(a+b)}{(x-b)}}{\frac{(a+b)}{(x+b)}}=\frac{-(a+b)}{(x-b)}\cdot\frac{(x+b)}{(a+b)}=\frac{-(x+b)}{(x-b)}$$

$$\frac{3bx-x^2}{bx-b^2}=\frac{3bx-x^2}{b(x-b)}$$

$$\frac{-(x+b)}{(x-b)}+\frac{3bx-x^2}{b(x-b)}=\frac{-b(x+b)+3bx-x^2}{b(x-b)}=\frac{-bx-b^2+3bx-x^2}{b(x-b)}=\frac{-b^2+2bx-x^2}{b(x-b)}=$$

$$\frac{-\left(b^2-2bx+x^2\right)}{b(x-b)}=\frac{-(b-x)(b-x)}{-b(b-x)}=\underline{\underline{\frac{(b-x)}{b}}}$$

2. $$\frac{a^2-2abc+(bc)^2}{\frac{a}{c}-\frac{a+ab}{a+c}}\cdot\left(\frac{1}{c}+\frac{b}{a-bc}\right)=$$

$$a^2-2abc+(bc)^2=(a-bc)(a-bc)$$

$$\frac{a}{c}-\frac{a+ab}{a+c}=\frac{a(a+c)-c(a+ab)}{c(a+c)}=\frac{a^2+ac-ac-abc}{c(a+c)}=\frac{a^2-abc}{c(a+c)}=\frac{a(a-bc)}{c(a+c)}$$

$$\frac{(a-bc)(a-bc)}{\frac{a(a-bc)}{c(a+c)}}=\frac{(a-bc)(a-bc)\cdot c(a+c)}{a(a-bc)}=\frac{(a-bc)\cdot c(a+c)}{a}$$

$$\left(\frac{1}{c}+\frac{b}{a-bc}\right)=\frac{a-bc+bc}{c(a-bc)}=\frac{a}{c(a-bc)}$$

$$\frac{(a-bc)\cdot c(a+c)}{a}\cdot\frac{a}{c(a-bc)}=\underline{\underline{a+c}}$$

3. $$\frac{4a+1}{(2a+1)^2} : \frac{2a+1-\frac{16a^2}{8a+4}}{2a+1-\frac{4a+2}{2a+1}} =$$

$$\frac{4a+1}{(2a+1)^2} = \frac{(4a+1)}{(2a+1)(2a+1)}$$

$$\frac{2a+1}{1} - \frac{16a^2}{4(2a+1)} = \frac{2a+1}{1} - \frac{4a^2}{(2a+1)} = \frac{(2a+1)(2a+1)-4a^2}{(2a+1)} = \frac{4a^2+4a+1-4a^2}{(2a+1)} =$$

$$\frac{4a+1}{(2a+1)} = \frac{(4a+1)}{(2a+1)}$$

$$2a+1-\frac{4a+2}{2a+1} = \frac{2a+1}{1} - \frac{4a+2}{(2a+1)} = \frac{(2a+1)(2a+1)-(4a+2)}{(2a+1)} =$$

$$\frac{4a^2+4a+1-4a-2}{(2a+1)} = \frac{4a^2-1}{(2a+1)} = \frac{(2a+1)(2a-1)}{(2a+1)} = (2a-1)$$

$$\frac{\frac{(4a+1)}{(2a+1)}}{\frac{(2a-1)}{1}} = \frac{(4a+1)}{(2a+1)(2a-1)}$$

$$\frac{(4a+1)}{(2a+1)(2a+1)} : \frac{(4a+1)}{(2a+1)(2a-1)} = \frac{(4a+1)(2a+1)(2a-1)}{(2a+1)(2a+1)(4a+1)} = \underline{\underline{\frac{(2a-1)}{(2a+1)}}}$$

4. $$\left[\left(\frac{a+x}{2x}\right)^2 - \left(\frac{a-x}{2x}\right)^2\right] - \frac{\frac{a+x}{x} - \frac{2x}{x-a}}{\frac{a^2+x^2}{x-a}} =$$

$$\left(\frac{a+x}{2x}\right)^2 - \left(\frac{a-x}{2x}\right)^2 = \frac{(a+x)(a+x)}{2x \cdot 2x} - \frac{(a-x)(a-x)}{2x \cdot 2x} = \frac{a^2+2ax+x^2-(a^2-2ax+x^2)}{4x^2} =$$

$$\frac{a^2+2ax+x^2-a^2+2ax-x^2}{4x^2} = \frac{4ax}{4x^2} = \frac{a}{x}$$

$$\frac{a+x}{x} - \frac{2x}{(x-a)} = \frac{(a+x)(x-a)-2x^2}{x(x-a)} = \frac{ax-a^2+x^2-ax-2x^2}{x(x-a)} = \frac{-a^2-x^2}{x(x-a)} = \frac{-(a^2+x^2)}{x(x-a)}$$

$$\frac{a^2+x^2}{x-a} = \frac{(a^2+x^2)}{(x-a)}$$

$$\frac{\frac{-(a^2+x^2)}{x(x-a)}}{\frac{(a^2+x^2)}{(x-a)}} = \frac{-(a^2+x^2)(x-a)}{x(x-a)(a^2+x^2)} = \frac{-1}{x}$$

$$\frac{a}{x} - \frac{-1}{x} = \frac{a}{x} + \frac{1}{x} = \underline{\underline{\frac{a+1}{x}}}$$

5. $\left[\frac{1}{\frac{a^2}{4}-b^2} : \left(\frac{5}{a+2b}+2\right)\right] \cdot \left[(a+2b+5)^2-(a+2b)^2\right] =$

$$\frac{a^2}{4}-\frac{b^2}{1}=\frac{a^2-4b^2}{4}=\frac{(a+2b)(a-2b)}{4}$$

$$\frac{1}{\frac{(a+2b)(a-2b)}{4}}=\frac{4}{(a+2b)(a-2b)}$$

$$\frac{5}{a+2b}+\frac{2}{1}=\frac{5+2(a+2b)}{(a+2b)}=\frac{5+2a+4b}{(a+2b)}=\frac{(2a+4b+5)}{(a+2b)}$$

$$\frac{4}{(a+2b)(a-2b)}:\frac{(2a+4b+5)}{(a+2b)}=\frac{4(a+2b)}{(a+2b)(a-2b)(2a+4b+5)}=\frac{4}{(a-2b)(2a+4b+5)}$$

$$(a+2b+5)^2-(a+2b)^2=(a+2b+5)(a+2b+5)-(a+2b)(a+2b)=$$

$$a^2+2ab+5a+2ab+4b^2+10b+5a+10b+25-(a^2+4ab+4b^2)=$$

$$a^2+2ab+5a+2ab+4b^2+10b+5a+10b+25-a^2-4ab-4b^2=$$

$$10a+20b+25=5(2a+4b+5)$$

$$\frac{4}{(a-2b)(2a+4b+5)}\cdot 5(2a+4b+5)=\frac{4\cdot 5(2a+4b+5)}{(a-2b)(2a+4b+5)}=\underline{\underline{\frac{20}{(a-2b)}}}$$

6. $\frac{1}{\frac{2}{m-1}+m+1}\cdot\frac{m-1-\frac{m(m-1)}{m+1}}{\frac{1}{m^2-1}-\frac{2m}{m^4-1}} =$

$$\frac{2}{(m-1)}+\frac{m+1}{1}=\frac{2+(m+1)(m-1)}{(m-1)}=\frac{2+m^2-1}{(m-1)}=\frac{m^2+1}{(m-1)}$$

$$\frac{1}{\frac{m^2+1}{(m-1)}}=\frac{(m-1)}{(m^2+1)}$$

$$\frac{m-1}{1}-\frac{m(m-1)}{(m+1)}=\frac{(m-1)(m+1)-m(m-1)}{(m+1)}=\frac{m^2-1-m^2+m}{(m+1)}=\frac{m-1}{(m+1)}=\frac{(m-1)}{(m+1)}$$

$$\frac{1}{(m^2-1)}-\frac{2m}{(m^2+1)(m^2-1)}=\frac{m^2+1-2m}{(m^2+1)(m+1)(m-1)}=\frac{m^2-2m+1}{(m^2+1)(m+1)(m-1)}=$$

$$\frac{(m-1)(m-1)}{(m^2+1)(m+1)(m-1)}=\frac{(m-1)}{(m^2+1)(m+1)}$$

$$\frac{\frac{(m-1)}{(m+1)}}{\frac{(m-1)}{(m^2+1)(m+1)}}=\frac{(m-1)}{(m+1)}\cdot\frac{(m^2+1)(m+1)}{(m-1)}=(m^2+1)$$

$$\frac{(m-1)}{(m^2+1)}\cdot(m^2+1)=\underline{\underline{m-1}}$$

7. $$\left(\frac{\frac{a-3b}{a^2-b^2}}{\frac{a}{a+b}}+\frac{\frac{3ab-b^2}{a^2-2ab+b^2}}{\frac{b^2}{a-b}}\right)\cdot\frac{1}{1+\frac{a}{b}}=$$

$$\frac{(a-3b)}{(a+b)(a-b)}\cdot\frac{(a+b)}{a}=\frac{(a-3b)}{a(a-b)}$$

$$\frac{3ab-b^2}{a^2-2ab+b^2}=\frac{b(3a-b)}{(a-b)(a-b)}$$

$$\frac{b^2}{a-b}=\frac{b^2}{(a-b)}$$

$$\frac{b(3a-b)}{(a-b)(a-b)}\cdot\frac{(a-b)}{b^2}=\frac{(3a-b)}{b(a-b)}$$

$$\frac{(a-3b)}{a(a-b)}+\frac{(3a-b)}{b(a-b)}=\frac{b(a-3b)+a(3a-b)}{ab(a-b)}=\frac{ab-3b^2+3a^2-ab}{ab(a-b)}=\frac{3a^2-3b^2}{ab(a-b)}=$$

$$\frac{3(a^2-b^2)}{ab(a-b)}=\frac{3(a+b)(a-b)}{ab(a-b)}=\frac{3(a+b)}{ab}$$

$$\frac{1}{1+\frac{a}{b}}=\frac{1}{\frac{b}{b}+\frac{a}{b}}=\frac{\frac{1}{1}}{\frac{a+b}{b}}=\frac{b}{(a+b)}$$

$$\frac{3(a+b)}{ab}\cdot\frac{b}{(a+b)}=\underline{\underline{\frac{3}{a}}}$$

8. $$\frac{\frac{r}{r+s}+\frac{s}{r-s}}{\frac{1}{(r+s)^2}+\frac{1}{(r-s)^2}}+(s^2-1)\cdot\left(\frac{s}{s+1}+\frac{1}{s-1}-\frac{1}{s^2-1}\right)=$$

$$\frac{r}{(r+s)}+\frac{s}{(r-s)}=\frac{r(r-s)+s(r+s)}{(r+s)(r-s)}=\frac{r^2-rs+rs+s^2}{(r+s)(r-s)}=\frac{r^2+s^2}{(r+s)(r-s)}=\frac{(r^2+s^2)}{(r+s)(r-s)}$$

$$\frac{1}{(r+s)(r+s)}+\frac{1}{(r-s)(r-s)}=\frac{(r-s)(r-s)+(r+s)(r+s)}{(r+s)(r+s)(r-s)(r-s)}=\frac{r^2-2rs+s^2+r^2+2rs+s^2}{(r+s)(r+s)(r-s)(r-s)}=$$

$$\frac{2r^2+2s^2}{(r+s)(r+s)(r-s)(r-s)}=\frac{2(r^2+s^2)}{(r+s)(r+s)(r-s)(r-s)}$$

$$\frac{(r^2+s^2)}{(r+s)(r-s)}\cdot\frac{(r+s)(r+s)(r-s)(r-s)}{2(r^2+s^2)}=\frac{(r+s)(r-s)}{2}$$

$$(s^2-1)=(s+1)(s-1)=\frac{(s+1)(s-1)}{1}$$

$$\frac{s}{(s+1)}+\frac{1}{(s-1)}-\frac{1}{(s+1)(s-1)}=\frac{s(s-1)+(s+1)-1}{(s+1)(s-1)}=\frac{s^2-s+s+1-1}{(s+1)(s-1)}=\frac{s^2}{(s+1)(s-1)}$$

$$\frac{(s+1)(s-1)}{1}\cdot\frac{s^2}{(s+1)(s-1)}=\frac{s^2}{1}$$

$$\frac{(r+s)(r-s)}{2}+\frac{s^2}{1}=\frac{r^2-s^2+2s^2}{2}=\underline{\underline{\frac{r^2+s^2}{2}}}$$

9. $\dfrac{\dfrac{b}{b^2-1}}{\dfrac{1}{b+1}-\dfrac{1}{b-1}} : \dfrac{\dfrac{2b}{b-3}-\dfrac{b}{b+4}}{1+\dfrac{23-b^2}{b^2+b-12}} =$

$$\frac{b}{b^2-1} = \frac{b}{(b+1)(b-1)}$$

$$\frac{1}{(b+1)} - \frac{1}{(b-1)} = \frac{(b-1)-(b+1)}{(b+1)(b-1)} = \frac{b-1-b-1}{(b+1)(b-1)} = \frac{-2}{(b+1)(b-1)}$$

$$\frac{b}{(b+1)(b-1)} \cdot \frac{(b+1)(b-1)}{-2} = \frac{b}{-2}$$

$$\frac{2b}{b-3} - \frac{b}{b+4} = \frac{2b(b+4)-b(b-3)}{(b-3)(b+4)} = \frac{2b^2+8b-b^2+3b}{(b-3)(b+4)} = \frac{b^2+11b}{(b-3)(b+4)} = \frac{b(b+11)}{(b-3)(b+4)}$$

$$\frac{1}{1} + \frac{23-b^2}{(b-3)(b+4)} = \frac{b^2+b-12+23-b^2}{(b-3)(b+4)} = \frac{b+11}{(b-3)(b+4)}$$

$$\frac{b(b+11)}{(b-3)(b+4)} \cdot \frac{(b-3)(b+4)}{(b+11)} = b$$

$$\frac{b}{-2} : b = \frac{b}{-2b} = \underline{\underline{-\frac{1}{2}}}$$

10. $\dfrac{(a+3)^2-(a+2)^2-5}{\frac{a^2-b^2}{a-b}} + \dfrac{\frac{(a-2b)^2-a^2}{2(b-a)}}{\frac{a^2+2a+ab+2b}{a+2}} =$

$(a+3)^2-(a+2)^2-5=(a+3)(a+3)-(a+2)(a+2)-5=$

$a^2+6a+9-(a^2+4a+4)-5=a^2+6a+9-a^2-4a-4-5=2a$

$\dfrac{a^2-b^2}{a-b}=\dfrac{(a+b)(a-b)}{(a-b)}=(a+b)$

$\dfrac{2a}{(a+b)}$

$\dfrac{(a-2b)^2-a^2}{2(b-a)}=\dfrac{(a-2b)(a-2b)-a^2}{2(b-a)}=\dfrac{a^2-4ab+4b^2-a^2}{2(b-a)}=\dfrac{-4ab+4b^2}{2(b-a)}=\dfrac{4b(b-a)}{2(b-a)}=$

$2b$

$\dfrac{a^2+2a+ab+2b}{a+2}=\dfrac{a(a+2)+b(a+2)}{(a+2)}=\dfrac{(a+b)(a+2)}{(a+2)}=(a+b)$

$\dfrac{2b}{(a+b)}$

$\dfrac{2a}{(a+b)}+\dfrac{2b}{(a+b)}=\dfrac{2a+2b}{(a+b)}=\dfrac{2(a+b)}{(a+b)}=\underline{\underline{2}}$

11. $$\frac{\frac{1}{1+\frac{x}{3}}+\frac{1}{1+\frac{3}{x}}}{\frac{1}{2}\cdot\frac{a-b}{ax+ay-b(x+y)}}=$$

$$\frac{1}{1+\frac{x}{3}}=\frac{1}{\frac{3}{3}+\frac{x}{3}}=\frac{1}{\frac{x+3}{3}}=\frac{3}{x+3}$$

$$\frac{1}{1+\frac{3}{x}}=\frac{1}{\frac{x}{x}+\frac{3}{x}}=\frac{1}{\frac{x+3}{x}}=\frac{x}{x+3}$$

$$\frac{3}{x+3}+\frac{x}{x+3}=\frac{x+3}{x+3}=1$$

$$a-b=(a-b)$$

$$ax+ay-b(x+y)=a(x+y)-b(x+y)=(a-b)(x+y)$$

$$\frac{1}{2}\cdot\frac{(a-b)}{(a-b)(x+y)}=\frac{1}{2(x+y)}$$

$$1:\frac{1}{2(x+y)}=\frac{2(x+y)}{1}=\underline{\underline{2(x+y)}}$$

12. $$\frac{\frac{2}{x-1}-\frac{1}{x-2}}{\frac{x-3}{x-\frac{4}{x}}}:\frac{x-5}{x^2-6x+5}=$$

$$\frac{2}{x-1}-\frac{1}{x-2}=\frac{2(x-2)-1(x-1)}{(x-1)(x-2)}=\frac{2x-4-x+1}{(x-1)(x-2)}=\frac{x-3}{(x-1)(x-2)}$$

$$\frac{x-3}{x-\frac{4}{x}}=\frac{(x-3)}{\frac{x^2}{x}-\frac{4}{x}}=\frac{(x-3)}{\frac{x^2-4}{x}}=\frac{(x-3)}{\frac{(x+2)(x-2)}{x}}=\frac{x(x-3)}{(x+2)(x-2)}$$

$$\frac{(x-3)}{(x-1)(x-2)}\cdot\frac{(x+2)(x-2)}{x(x-3)}=\frac{(x+2)}{x(x-1)}$$

$$\frac{x-5}{x^2-6x+5}=\frac{(x-5)}{(x-5)(x-1)}=\frac{1}{(x-1)}$$

$$\frac{(x+2)}{x(x-1)}\cdot\frac{(x-1)}{1}=\underline{\underline{\frac{x+2}{x}}}$$

13. $\frac{ax+3a^2}{x^2+6ax+9a^2} + \frac{\left(x+\frac{1}{2}\right)^2-\left(x-\frac{1}{2}\right)^2+5a}{\frac{x^2-9a^2}{x-3a}} =$

$$\frac{ax+3a^2}{x^2+6ax+9a^2} = \frac{a(x+3a)}{(x+3a)(x+3a)} = \frac{a}{(x+3a)}$$

$$\left(x+\frac{1}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2 + 5a = \left(x+\frac{1}{2}\right)\left(x+\frac{1}{2}\right) - \left(x-\frac{1}{2}\right)\left(x-\frac{1}{2}\right) + 5a =$$

$$x^2 + x + \frac{1}{4} - \left(x^2 - x + \frac{1}{4}\right) + 5a = x^2 + x + \frac{1}{4} - x^2 + x - \frac{1}{4} + 5a =$$

$$= 2x + 5a$$

$$\frac{x^2-9a^2}{x-3a} = \frac{(x+3a)(x-3a)}{(x-3a)} = (x+3a)$$

$$\frac{a}{(x+3a)} + \frac{2x+5a}{(x+3a)} = \frac{a+2x+5a}{(x+3a)} = \frac{2x+6a}{(x+3a)} = \frac{2(x+3a)}{(x+3a)} = \underline{\underline{2}}$$

14. $\frac{\frac{14x^3-2x^2y}{7x-y}}{\frac{x^2-y^2}{x(y+1)-y(x+1)}} - \frac{y^2}{\frac{x^2+2xy+y^2}{2x+2y}} =$

$$\frac{14x^3-2x^2y}{7x-y} = \frac{2x^2(7x-y)}{(7x-y)} = 2x^2$$

$$\frac{x^2-y^2}{x(y+1)-y(x+1)} = \frac{(x+y)(x-y)}{xy+x-xy-y} = \frac{(x+y)(x-y)}{(x-y)} = (x+y)$$

$y^2$

$$\frac{x^2+2xy+y^2}{2x+2y} = \frac{(x+y)(x+y)}{2(x+y)} = \frac{(x+y)}{2}$$

$$y^2 \cdot \frac{2}{(x+y)} = \frac{2y^2}{(x+y)}$$

$$\frac{2x^2}{(x+y)} - \frac{2y^2}{(x+y)} = \frac{2(x^2-y^2)}{(x+y)} = \frac{2(x+y)(x-y)}{(x+y)} = \underline{\underline{2(x-y)}}$$

15. $$\frac{\frac{x^3-12x^2+35x}{x^3-2x^2-35x}}{-\frac{21}{4}+\left(x+\frac{1}{2}\right)^2-x^2}+\frac{5-x}{x^2-25}=$$

$$\frac{x^3-12x^2+35x}{x^3-2x^2-35x}=\frac{x(x^2-12x+35)}{x(x^2-2x-35)}=\frac{x(x-5)(x-7)}{x(x-7)(x+5)}=\frac{(x-5)}{(x+5)}$$

$$-\frac{21}{4}+\left(x+\frac{1}{2}\right)^2-x^2=-\frac{21}{4}+\left(x+\frac{1}{2}\right)\left(x+\frac{1}{2}\right)-x^2=$$

$$-\frac{21}{4}+x^2+x+\frac{1}{4}-x^2=x-\frac{20}{4}=x-5=\frac{(x-5)}{1}$$

$$\frac{(x-5)}{(x+5)}\cdot\frac{1}{(x-5)}=\frac{1}{(x+5)}$$

$$\frac{5-x}{x^2-25}=\frac{(5-x)}{(x+5)(x-5)}=\frac{-(x-5)}{(x+5)(x-5)}=\frac{-1}{(x+5)}$$

$$\frac{1}{(x+5)}+\frac{-1}{(x+5)}=\frac{1-1}{(x+5)}=\frac{0}{(x+5)}=\underline{\underline{0}}$$

16. $$\frac{\left(x+\frac{1}{2}\right)(2x+1)-\frac{1}{2}}{\frac{x(a+z)+3a+3z}{a+z}}:\frac{(x^2-1)x}{x^2+2x-3}=$$

$$\left(x+\frac{1}{2}\right)(2x+1)-\frac{1}{2}=2x^2+x+x+\frac{1}{2}-\frac{1}{2}=2x^2+2x=2x(x+1)$$

$$\frac{x(a+z)+3a+3z}{a+z}=\frac{x(a+z)+3(a+z)}{(a+z)}=\frac{(x+3)(a+z)}{(a+z)}=(x+3)$$

$$\frac{2x(x+1)}{(x+3)}$$

$$(x^2-1)x=x(x+1)(x-1)$$

$$x^2+2x-3=(x+3)(x-1)$$

$$\frac{x(x+1)(x-1)}{(x+3)(x-1)}=\frac{x(x+1)}{(x+3)}$$

$$\frac{2x(x+1)}{(x+3)}\cdot\frac{(x+3)}{x(x+1)}=\underline{\underline{2}}$$