

$$\begin{aligned}
 1. \quad & \frac{\frac{4a-6b}{3b}}{\frac{2a}{3} - \frac{2a-b}{2}} = \\
 & \frac{2(2a-3b)}{3b} \\
 & \frac{2a}{3} - \frac{2a-b}{2} = \frac{4a-3(2a-b)}{6} = \frac{4a-6a+3b}{6} = \frac{(3b-2a)}{6} = \frac{-1(2a-3b)}{6} \\
 & \frac{2(2a-3b)}{3b} \cdot \frac{6}{-1(2a-3b)} = \underline{\underline{\frac{4}{b}}}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{\frac{x+y}{6x-2y}}{\frac{2}{3x-y} - \frac{1}{2x}} = \\
 & \frac{x+y}{6x-2y} = \frac{(x+y)}{2(3x-y)} \\
 & \frac{2}{3x-y} - \frac{1}{2x} = \frac{4x-(3x-y)}{2x(3x-y)} = \frac{4x-3x+y}{2x(3x-y)} = \frac{x+y}{2x(3x-y)} = \frac{(x+y)}{2x(3x-y)} \\
 & \frac{(x+y)}{2(3x-y)} \cdot \frac{2x(3x-y)}{(x+y)} = \underline{\underline{x}}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \frac{1 + \frac{(a-b)^2}{4ab}}{1 + \frac{a}{b}} = \\
 & \frac{4ab + a^2 - 2ab + b^2}{4ab} = \frac{a^2 + 2ab + b^2}{4ab} = \frac{(a+b)(a+b)}{4ab} \\
 & 1 + \frac{a}{b} = \frac{b+a}{b} = \frac{(a+b)}{b} \\
 & \frac{(a+b)(a+b)}{4ab} \cdot \frac{b}{(a+b)} = \underline{\underline{\frac{a+b}{4a}}}
 \end{aligned}$$

$$4. \frac{x^3+7x^2-8x}{x^3+6x^2-16x} \cdot \left(\frac{1}{x} - \frac{2}{x+2}\right) =$$

$$\frac{x(x^2+7x-8)}{x(x^2+6x-16)} = \frac{x(x+8)(x-1)}{x(x+8)(x-2)} = \frac{(x-1)}{(x-2)}$$

$$\left(\frac{1}{x} - \frac{2}{x+2}\right) = \frac{x+2-2x}{x(x+2)} = \frac{2-x}{x(x+2)} = \frac{-(x-2)}{x(x+2)}$$

$$\frac{(x-1)}{(x-2)} \cdot \frac{-(x-2)}{x(x+2)} = \frac{-(x-1)}{x(x+2)} = \frac{(1-x)}{x(x+2)}$$

$$5. \left(\frac{x-2y}{x+y} - \frac{x-2y}{x-y}\right) \cdot \frac{6x^2-6y^2}{(x-3y)(x-2y)} =$$

$$\left(\frac{x-2y}{x+y} - \frac{x-2y}{x-y}\right) = \frac{(x-2y)(x-y) - (x-2y)(x+y)}{(x+y)(x-y)} =$$

$$\frac{x^2-xy-2xy+2y^2 - (x^2+xy-2xy-2y^2)}{(x+y)(x-y)} = \frac{x^2-xy-2xy+2y^2 - x^2-xy+2xy+2y^2}{(x+y)(x-y)} =$$

$$\frac{4y^2-2xy}{(x+y)(x-y)} = \frac{2y(2y-x)}{(x+y)(x-y)}$$

$$\frac{6x^2-6y^2}{(x-3y)(x-2y)} = \frac{6(x^2-y^2)}{(x-3y)(x-2y)} = \frac{6(x+y)(x-y)}{(x-3y)(x-2y)}$$

$$\frac{2y(2y-x)}{(x+y)(x-y)} \cdot \frac{6(x+y)(x-y)}{(x-3y)(x-2y)} = \frac{-2y(x-2y)}{(x+y)(x-y)} \cdot \frac{6(x+y)(x-y)}{(x-3y)(x-2y)} = \frac{-12y}{(x-3y)} = \frac{12y}{(3y-x)}$$

$$6. \frac{\frac{2a}{a-4} + \frac{3a}{3-a}}{\frac{a^2-36}{a^2-7a+12}} =$$

$$\frac{2a}{a-4} + \frac{3a}{3-a} = \frac{2a(3-a)+3a(a-4)}{(a-4)(3-a)} = \frac{6a-2a^2+3a^2-12a}{(a-4)(3-a)} = \frac{a^2-6a}{(a-4)(3-a)} =$$

$$\frac{a(a-6)}{(a-4)(3-a)} = \frac{-a(a-6)}{(a-4)(a-3)}$$

$$\frac{a^2-36}{a^2-7a+12} = \frac{(a+6)(a-6)}{(a-4)(a-3)}$$

$$\frac{-a(a-6)}{(a-4)(a-3)} \cdot \frac{(a-4)(a-3)}{(a+6)(a-6)} = \frac{-a}{(a+6)}$$

$$7. \quad \frac{1 + \frac{1}{a} - \frac{2}{a^2}}{a-1} =$$

$$1 + \frac{1}{a} - \frac{2}{a^2} = \frac{a^2 + a - 2}{a^2} = \frac{(a-1)(a+2)}{a^2}$$

$$a - 1 = \frac{(a-1)}{1}$$

$$\frac{(a-1)(a+2)}{a^2} \cdot \frac{1}{(a-1)} = \underline{\underline{\frac{(a+2)}{a^2}}}$$

$$8. \quad \frac{\frac{36x^2 - 16y^2}{3x+2y}}{\frac{3x-2y}{6x+4y}} =$$

$$\frac{36x^2 - 16y^2}{3x+2y} = \frac{4(9x^2 - 4y^2)}{(3x+2y)} = \frac{4(3x+2y)(3x-2y)}{(3x+2y)} = \frac{4(3x-2y)}{1}$$

$$\frac{3x-2y}{6x+4y} = \frac{(3x-2y)}{2(3x+2y)}$$

$$\frac{4(3x-2y)}{1} \cdot \frac{2(3x+2y)}{(3x-2y)} = \underline{\underline{8(3x+2y)}}$$

$$9. \quad \frac{\frac{18a^2b + 6ab^2}{a+b} - \frac{6a^2b - 6ab^2}{a}}{a-b} =$$

$$\frac{18a^2b + 6ab^2}{6a^2b - 6ab^2} = \frac{6ab(3a+b)}{6ab(a-b)} = \frac{(3a+b)}{(a-b)}$$

$$\frac{a+b}{a-b} - \frac{a}{a+b} = \frac{(a+b)(a+b) - a(a-b)}{(a+b)(a-b)} = \frac{a^2 + 2ab + b^2 - a^2 + ab}{(a+b)(a-b)} = \frac{b^2 + 3ab}{(a+b)(a-b)} =$$

$$\frac{b(b+3a)}{(a+b)(a-b)} = \frac{b(3a+b)}{(a+b)(a-b)}$$

$$\frac{(3a+b)}{(a-b)} \cdot \frac{(a+b)(a-b)}{b(3a+b)} = \underline{\underline{\frac{(a+b)}{b}}}$$

$$10. \frac{\frac{2ab-3a^2}{6a+9b}}{\frac{9a^3-12a^2b+4ab^2}{4a^2-9b^2}} =$$

$$\frac{2ab-3a^2}{6a+9b} = \frac{a(2b-3a)}{3(2a+3b)} = \frac{-a(3a-2b)}{3(2a+3b)}$$

$$\frac{9a^3-12a^2b+4ab^2}{4a^2-9b^2} = \frac{a(9a^2-12ab+4b^2)}{(2a+3b)(2a-3b)} = \frac{a(3a-2b)(3a-2b)}{(2a+3b)(2a-3b)}$$

$$\frac{-a(3a-2b)}{3(2a+3b)} \cdot \frac{(2a+3b)(2a-3b)}{a(3a-2b)(3a-2b)} = \frac{-(2a-3b)}{3(3a-2b)} = \frac{(3b-2a)}{3(3a-2b)}$$

$$11. \frac{1-\frac{1}{a-1}}{\frac{2}{a}-1} - \frac{1-\frac{1}{a-1}}{2-a} =$$

$$1 - \frac{1}{a-1} = \frac{a-1-1}{(a-1)} = \frac{(a-2)}{(a-1)}$$

$$\frac{2}{a} - 1 = \frac{2-a}{a} = \frac{-(a-2)}{a}$$

$$\frac{(a-2)}{(a-1)} \cdot \frac{a}{-(a-2)} = \frac{a}{-(a-1)} = \frac{-a}{(a-1)}$$

$$1 - \frac{1}{a-1} = \frac{a-1-1}{(a-1)} = \frac{(a-2)}{(a-1)}$$

$$2 - a = \frac{2-a}{1} = \frac{-(a-2)}{1}$$

$$\frac{(a-2)}{(a-1)} \cdot \frac{1}{-(a-2)} = \frac{1}{-(a-1)} = \frac{-1}{(a-1)}$$

$$\frac{-a}{(a-1)} - \frac{-1}{(a-1)} = \frac{-a+1}{(a-1)} = \frac{1-a}{(a-1)} = \frac{-1(a-1)}{(a-1)} = \underline{\underline{-1}}$$

$$12. \quad 1 - \frac{1-a}{1+a-\frac{4a}{1+a}} =$$

$$1 - a = \frac{(1-a)}{1} = \frac{-(a-1)}{1}$$

$$1 + a - \frac{4a}{1+a} = \frac{1+a+a(1+a)-4a}{(1+a)} = \frac{1+a+a+a^2-4a}{(1+a)} = \frac{a^2-2a+1}{(1+a)} = \frac{(a-1)(a-1)}{(1+a)}$$

$$\frac{-(a-1)}{1} \cdot \frac{(1+a)}{(a-1)(a-1)} = \frac{-(1+a)}{(a-1)}$$

$$1 - \frac{-(1+a)}{(a-1)} = \frac{a-1+(1+a)}{(a-1)} = \frac{a-1+1+a}{(a-1)} = \frac{2a}{(a-1)}$$

$$13. \quad \frac{\frac{1}{1-x} - \frac{2x}{1-x^2}}{\frac{1}{1+x} - 1} =$$

$$\frac{1}{1-x} - \frac{2x}{1-x^2} = \frac{1}{(1-x)} - \frac{2x}{(1+x)(1-x)} = \frac{1+x-2x}{(1+x)(1-x)} = \frac{1-x}{(1+x)(1-x)} = \frac{1}{(1+x)}$$

$$\frac{1}{1+x} - 1 = \frac{1-(1+x)}{(1+x)} = \frac{1-1-x}{(1+x)} = \frac{-x}{(1+x)}$$

$$\frac{1}{(1+x)} \cdot \frac{(1+x)}{-x} = \frac{1}{-x}$$

$$14. \quad \frac{\frac{1}{x} + \frac{1}{a-x}}{\frac{1}{a} - \frac{1}{a-x}} =$$

$$\frac{1}{x} + \frac{1}{a-x} = \frac{a-x+x}{x(a-x)} = \frac{a}{x(a-x)}$$

$$\frac{1}{a} - \frac{1}{a-x} = \frac{a-x-a}{a(a-x)} = \frac{-x}{a(a-x)}$$

$$\frac{a}{x(a-x)} \cdot \frac{a(a-x)}{-x} = \frac{a^2}{-x^2} = \frac{a^2}{x^2}$$

$$15. \frac{\frac{1}{1+x} - \frac{1}{1-x}}{\frac{1}{1+\frac{1}{x}} + \frac{1}{1-\frac{1}{x}}} =$$

$$\frac{1}{1+x} - \frac{1}{1-x} = \frac{1-x-(1+x)}{(1+x)(1-x)} = \frac{1-x-1-x}{(1+x)(1-x)} = \frac{-2x}{(1+x)(1-x)} = \frac{2x}{(x+1)(x-1)}$$

$$\frac{1}{1+\frac{1}{x}} = \frac{1}{\frac{x+1}{x}} = \frac{\frac{1}{1}}{\frac{x+1}{x}} = \frac{x}{x+1} = \frac{x}{(x+1)}$$

$$\frac{1}{1-\frac{1}{x}} = \frac{1}{\frac{x-1}{x}} = \frac{\frac{1}{1}}{\frac{x-1}{x}} = \frac{x}{x-1} = \frac{x}{(x-1)}$$

$$\frac{x}{(x+1)} + \frac{x}{(x-1)} = \frac{x(x-1)+x(x+1)}{(x+1)(x-1)} = \frac{x^2-x+x^2+x}{(x+1)(x-1)} = \frac{2x^2}{(x+1)(x-1)}$$

$$\frac{2x}{(x+1)(x-1)} \cdot \frac{(x+1)(x-1)}{2x^2} = \frac{1}{x}$$

$$16. (a^2 - 3a + 2) \cdot \frac{\frac{1}{a-1} + \frac{1}{a+1}}{a^2 - 2a} =$$

$$(a^2 - 3a + 2) = (a - 2)(a - 1) = \frac{(a-2)(a-1)}{1}$$

$$\frac{1}{a-1} + \frac{1}{a+1} = \frac{a+1+a-1}{(a-1)(a+1)} = \frac{2a}{(a-1)(a+1)}$$

$$a^2 - 2a = a(a - 2) = \frac{a(a-2)}{1}$$

$$\frac{(a-2)(a-1)}{1} \cdot \frac{2a}{(a-1)(a+1)} \cdot \frac{1}{a(a-2)} = \frac{2}{(a+1)}$$

$$17. \frac{m + \frac{mn}{m-n}}{mn - \frac{mn^2}{m+n}} - \frac{1}{n} =$$

$$m + \frac{mn}{m-n} = \frac{m(m-n) + mn}{(m-n)} = \frac{m^2 - mn + mn}{(m-n)} = \frac{m^2}{(m-n)}$$

$$mn - \frac{mn^2}{m+n} = \frac{mn(m+n) - mn^2}{(m+n)} = \frac{m^2n + mn^2 - mn^2}{(m+n)} = \frac{m^2n}{(m+n)}$$

$$\frac{m^2}{(m-n)} \cdot \frac{(m+n)}{m^2n} = \frac{(m+n)}{n(m-n)}$$

$$\frac{(m+n)}{n(m-n)} - \frac{1}{n} = \frac{m+n - (m-n)}{n(m-n)} = \frac{m+n-m+n}{n(m-n)} = \frac{2n}{n(m-n)} = \frac{2}{\underline{\underline{(m-n)}}$$

$$18. a - \frac{-1}{1-2a + \frac{3a^2}{1+2a}} =$$

$$a = \frac{a}{1}$$

$$-1 = \frac{-1}{1}$$

$$1 - 2a + \frac{3a^2}{1+2a} = \frac{(1-2a)(1+2a) + 3a^2}{(1+2a)} = \frac{1-4a^2+3a^2}{(1+2a)} = \frac{1-a^2}{(1+2a)} = \frac{(1+a)(1-a)}{(1+2a)}$$

$$\frac{-1}{1} \cdot \frac{(1+2a)}{(1+a)(1-a)} = \frac{-1(1+2a)}{(1+a)(1-a)}$$

$$\frac{a}{1} - \frac{-1(1+2a)}{(1+a)(1-a)} = \frac{a}{1} + \frac{1(1+2a)}{(1+a)(1-a)} = \frac{a(1-a^2) + (1+2a)}{(1+a)(1-a)} =$$

$$\frac{a(1+a)(1-a) + (1+2a)}{(1+a)(1-a)} = \frac{a-a^3+1+2a}{(1+a)(1-a)} = \frac{-a^3+3a+1}{\underline{\underline{(1+a)(1-a)}}$$

$$19. \frac{\frac{4a^2}{b^2} - 9}{\frac{a^2}{b} \cdot \left(\frac{4}{b} + \frac{12}{a}\right) + 9} =$$

$$\frac{4a^2}{b^2} - 9 = \frac{4a^2 - 9b^2}{b^2} = \frac{(2a+3b)(2a-3b)}{b^2}$$

$$\frac{a^2}{b} \cdot \left(\frac{4}{b} + \frac{12}{a}\right) + 9 = \frac{a^2}{b} \cdot \left(\frac{4a+12b}{ab}\right) + 9 = \frac{a^2(4a+12b)}{b \cdot ab} + 9 =$$

$$\frac{a(4a+12b)}{b^2} + \frac{9b^2}{b^2} = \frac{4a^2+12ab+9b^2}{b^2} = \frac{(2a+3b)(2a+3b)}{b^2}$$

$$\frac{(2a+3b)(2a-3b)}{b^2} \cdot \frac{b^2}{(2a+3b)(2a+3b)} = \frac{(2a-3b)}{(2a+3b)}$$

$$20. \frac{\frac{6a^3+18a^2b+12ab^2}{a+2b} + b^2 - ab}{3a+b} =$$

$$\frac{6a^3+18a^2b+12ab^2}{a+2b} = \frac{6a(a^2+3ab+2b^2)}{(a+2b)} = \frac{6a(a+b)(a+2b)}{(a+2b)} = 6a(a+b)$$

$$6a(a+b) + b^2 - ab = 6a^2 + 6ab + b^2 - ab = 6a^2 + 5ab + b^2 = (3a+b)(2a+b)$$

$$\frac{(3a+b)(2a+b)}{3a+b} = \underline{\underline{2a+b}}$$

$$21. \frac{2a + \frac{a}{a-1}}{1 + a - \frac{3a^2}{1-a}} + 1 =$$

$$2a + \frac{a}{a-1} = \frac{2a(a-1)+a}{(a-1)} = \frac{2a^2-2a+a}{(a-1)} = \frac{2a^2-a}{(a-1)} = \frac{a(2a-1)}{(a-1)}$$

$$1 + a - \frac{3a^2}{1-a} = \frac{(1+a)(1-a)-3a^2}{(1-a)} = \frac{1-a^2-3a^2}{(1-a)} = \frac{1-4a^2}{(1-a)} = \frac{(1-2a)(1+2a)}{(1-a)}$$

$$\frac{a(2a-1)}{(a-1)} \cdot \frac{(1-a)}{(1-2a)(1+2a)} = \frac{a(1-2a)}{(1-a)} \cdot \frac{(1-a)}{(1-2a)(1+2a)} = \frac{a}{(1+2a)}$$

$$\frac{a}{(1+2a)} + 1 = \frac{a+1+2a}{(1+2a)} = \underline{\underline{\frac{3a+1}{2a+1}}}$$