

### 3. Bez

### Bruchterme + / -

### Lösungen AB 6

$$1. \quad \frac{2a}{(a-2x)(a+2x)} - \frac{1}{a-2x} = \frac{2a-1(a+2x)}{(a-2x)(a+2x)} = \frac{2a-a-2x}{(a-2x)(a+2x)} = \frac{a-2x}{(a-2x)(a+2x)} = \frac{1}{(a+2x)}$$

$$2. \quad \frac{6m}{9m^2-1} - \frac{1}{3m-1} = \frac{6m}{(3m+1)(3m-1)} - \frac{1}{(3m-1)} = \frac{6m-1(3m+1)}{(3m+1)(3m-1)} = \frac{6m-3m-1}{(3m+1)(3m-1)} = \frac{3m-1}{(3m+1)(3m-1)} = \frac{1}{(3m+1)}$$

$$3. \quad \frac{x}{ax-a} + \frac{1-x^2}{ax^2-a} = \frac{x}{a(x-1)} + \frac{1-x^2}{a(x^2-1)} = \frac{x}{a(x-1)} + \frac{1-x^2}{a(x+1)(x-1)} = \frac{x(x+1)+1-x^2}{a(x+1)(x-1)} = \frac{x^2+x+1-x^2}{a(x+1)(x-1)} = \frac{x+1}{a(x+1)(x-1)} = \frac{1}{a(x-1)}$$

$$4. \quad \frac{1}{a-3} - \frac{5}{a^2-a-6} = \frac{1}{(a-3)} - \frac{5}{(a-3)(a+2)} = \frac{a+2-5}{(a-3)(a+2)} = \frac{a-3}{(a-3)(a+2)} = \frac{1}{(a+2)}$$

$$5. \quad \frac{3x-8}{x^2-6x+8} - \frac{2}{x-2} = \frac{3x-8}{(x-2)(x-4)} - \frac{2}{(x-2)} = \frac{3x-8-2(x-4)}{(x-2)(x-4)} = \frac{3x-8-2x+8}{(x-2)(x-4)} = \frac{x}{(x-2)(x-4)}$$

$$6. \quad \frac{2m-2}{m^2-2m-15} - \frac{1}{m-5} = \frac{2m-2}{(m-5)(m+3)} - \frac{1}{(m-5)} = \frac{2m-2-1(m+3)}{(m-5)(m+3)} = \frac{2m-2-m-3}{(m-5)(m+3)} = \frac{m-5}{(m-5)(m+3)} = \frac{1}{(m+3)}$$

$$7. \quad \frac{m+2}{m^2-10m+24} + \frac{3}{m-4} = \frac{m+2}{(m-4)(m-6)} + \frac{3}{(m-4)} = \frac{m+2+3(m-6)}{(m-4)(m-6)} = \frac{m+2+3m-18}{(m-4)(m-6)} = \frac{4m-16}{(m-4)(m-6)} = \frac{4(m-4)}{(m-4)(m-6)} = \frac{4}{(m-6)}$$

$$8. \frac{3}{x-y} + \frac{y-3x}{x^2-y^2} = \frac{3}{(x-y)} + \frac{y-3x}{(x+y)(x-y)} = \frac{3(x+y)+y-3x}{(x+y)(x-y)} = \frac{3x+3y+y-3x}{(x+y)(x-y)} = \frac{4y}{(x+y)(x-y)}$$

$$9. \frac{x}{x-y} - \frac{y}{x+y} - \frac{2y^2}{x^2-y^2} = \frac{x}{(x-y)} - \frac{y}{(x+y)} - \frac{2y^2}{(x+y)(x-y)} = \frac{x(x+y)-y(x-y)-2y^2}{(x+y)(x-y)} = \frac{x^2+xy-xy+y^2-2y^2}{(x+y)(x-y)} = \frac{x^2-y^2}{(x+y)(x-y)} = \frac{(x+y)(x-y)}{(x+y)(x-y)} = \underline{\underline{1}}$$

$$10. \frac{x}{x+y} + \frac{y}{x-y} - \frac{x^2}{x^2-y^2} = \frac{x}{(x+y)} + \frac{y}{(x-y)} - \frac{x^2}{(x+y)(x-y)} = \frac{x(x-y)+y(x+y)-x^2}{(x+y)(x-y)} = \frac{x^2-xy+xy+y^2-x^2}{(x+y)(x-y)} = \frac{y^2}{(x+y)(x-y)}$$

$$11. \frac{2a}{a-3} + \frac{3a^2-20a-3}{a^2-9} - \frac{5a}{a+3} = \frac{2a}{(a-3)} + \frac{3a^2-20a-3}{(a+3)(a-3)} - \frac{5a}{(a+3)} = \frac{2a(a+3)+3a^2-20a-3-5a(a-3)}{(a+3)(a-3)} = \frac{2a^2+6a+3a^2-20a-3-5a^2+15a}{(a+3)(a-3)} = \frac{a-3}{(a+3)(a-3)} = \underline{\underline{\frac{1}{(a+3)}}}$$

$$12. \frac{4a^2-5a+3}{a^2-9} - \frac{3a}{a+3} - \frac{a}{a-3} = \frac{4a^2-5a+3}{(a+3)(a-3)} - \frac{3a}{(a+3)} - \frac{a}{(a-3)} = \frac{4a^2-5a+3-3a(a-3)-a(a+3)}{(a+3)(a-3)} = \frac{4a^2-5a+3-3a^2+9a-a^2-3a}{(a+3)(a-3)} = \frac{a+3}{(a+3)(a-3)} = \underline{\underline{\frac{1}{(a-3)}}}$$

$$13. \frac{2-2a}{2a+6} - \frac{6a-2}{a^2+a-6} + 1 = \frac{2-2a}{2(a+3)} - \frac{6a-2}{(a+3)(a-2)} + \frac{a^2+a-6}{(a+3)(a-2)} = \frac{(2-2a)(a-2)-2(6a-2)+2(a^2+a-6)}{2(a+3)(a-2)} = \frac{2a-4-2a^2+4a-12a+4+2a^2+2a-12}{2(a+3)(a-2)} = \frac{-4a-12}{2(a+3)(a-2)} = \frac{-4(a+3)}{2(a+3)(a-2)} = \underline{\underline{\frac{-2}{(a-2)}}}$$

$$\begin{aligned}
14. \quad & \frac{a}{a-2} - \frac{a+4}{a+2} - \frac{10-a}{a^2-4} = \frac{a}{(a-2)} - \frac{a+4}{(a+2)} - \frac{10-a}{(a+2)(a-2)} = \\
& \frac{a(a+2) - (a+4)(a-2) - (10-a)}{(a+2)(a-2)} = \frac{a^2+2a - (a^2+4a-2a-8) - 10+a}{(a+2)(a-2)} = \\
& \frac{a^2+2a - a^2 - 4a + 2a + 8 - 10 + a}{(a+2)(a-2)} = \frac{a-2}{(a+2)(a-2)} = \underline{\underline{\frac{1}{(a+2)}}}
\end{aligned}$$

$$\begin{aligned}
15. \quad & \frac{2a^2-9a+11}{a^2-5a+6} - \frac{2a-4}{a-3} + \frac{2a+5}{a^2-2a} = \frac{2a^2-9a+11}{(a-3)(a-2)} - \frac{2a-4}{(a-3)} + \frac{2a+5}{a(a-2)} = \\
& \frac{a(2a^2-9a+11) - a(2a-4)(a-2) + (2a+5)(a-3)}{a(a-3)(a-2)} = \\
& \frac{2a^3-9a^2+11a - a(2a^2-4a-4a+8) + 2a^2-6a+5a-15}{a(a-3)(a-2)} = \\
& \frac{2a^3-9a^2+11a - 2a^3+4a^2+4a^2-8a+2a^2-6a+5a-15}{a(a-3)(a-2)} = \frac{a^2+2a-15}{a(a-3)(a-2)} = \\
& \frac{(a+5)(a-3)}{a(a-3)(a-2)} = \underline{\underline{\frac{(a+5)}{a(a-2)}}}
\end{aligned}$$

$$\begin{aligned}
16. \quad & \frac{a-2}{2a+6} - \frac{a}{a-2} + \frac{6a-2}{a^2+a-6} = \frac{a-2}{2(a+3)} - \frac{a}{a-2} + \frac{6a-2}{(a+3)(a-2)} = \\
& \frac{(a-2)(a-2) - 2a(a+3) + 2(6a-2)}{2(a+3)(a-2)} = \frac{a^2-4a+4 - 2a^2-6a+12a-4}{2(a+3)(a-2)} = \frac{-a^2+2a}{2(a+3)(a-2)} = \\
& \frac{-a(a-2)}{2(a+3)(a-2)} = \underline{\underline{\frac{-a}{2(a+3)}}}
\end{aligned}$$

$$\begin{aligned}
17. \quad & \frac{2a+b}{a-b} - \frac{a-2b}{a+b} + \frac{2b(b-2a)}{a^2-b^2} = \frac{2a+b}{(a-b)} - \frac{a-2b}{(a+b)} + \frac{2b(b-2a)}{(a+b)(a-b)} = \\
& \frac{(2a+b)(a+b) - (a-2b)(a-b) + 2b(b-2a)}{(a+b)(a-b)} = \\
& \frac{2a^2+ab+2ab+b^2 - (a^2-ab-2ab+2b^2) + 2b^2-4ab}{(a+b)(a-b)} = \\
& \frac{2a^2+ab+2ab+b^2 - a^2+ab+2ab-2b^2+2b^2-4ab}{(a+b)(a-b)} = \frac{a^2+2ab+b^2}{(a+b)(a-b)} = \frac{(a+b)(a+b)}{(a+b)(a-b)} = \\
& \underline{\underline{\frac{(a+b)}{(a-b)}}}
\end{aligned}$$

$$18. \frac{3ab+b^2}{2a-5b} + \frac{b}{2} + \frac{17b^2}{10b-4a} = \frac{3ab+b^2}{(2a-5b)} + \frac{b}{2} + \frac{17b^2}{-2(2a-5b)} =$$

$$\frac{3ab+b^2}{(2a-5b)} + \frac{b}{2} + \frac{-17b^2}{2(2a-5b)} = \frac{2(3ab+b^2)+b(2a-5b)-17b^2}{2(2a-5b)} =$$

$$\frac{6ab+2b^2+2ab-5b^2-17b^2}{2(2a-5b)} = \frac{8ab-20b^2}{2(2a-5b)} = \frac{4b(2a-5b)}{2(2a-5b)} = \underline{\underline{2b}}$$

$$19. \frac{3}{a+2} - \frac{4}{a+5} - \frac{5-2a}{a^2+7a+10} = \frac{3}{(a+2)} - \frac{4}{(a+5)} - \frac{5-2a}{(a+2)(a+5)} =$$

$$\frac{3(a+5)-4(a+2)-(5-2a)}{(a+2)(a+5)} = \frac{3a+15-4a-8-5+2a}{(a+2)(a+5)} = \frac{a+2}{(a+2)(a+5)} = \underline{\underline{\frac{1}{(a+5)}}}$$

$$20. \frac{9+11a-3a^2}{a^2-7a+12} + \frac{5a}{a-3} - \frac{2a}{a-4} = \frac{9+11a-3a^2}{(a-4)(a-3)} + \frac{5a}{(a-3)} - \frac{2a}{(a-4)} =$$

$$\frac{9+11a-3a^2+5a(a-4)-2a(a-3)}{(a-4)(a-3)} = \frac{9+11a-3a^2+5a^2-20a-2a^2+6a}{(a-4)(a-3)} =$$

$$\frac{-3a+9}{(a-4)(a-3)} = \frac{-3(a-3)}{(a-4)(a-3)} = \underline{\underline{\frac{-3}{(a-4)}}}$$

$$21. \frac{2a+b}{4a-2b} + \frac{4a^2-4ab-b^2}{4a^2-b^2} - 1 = \frac{2a+b}{2(2a-b)} + \frac{4a^2-4ab-b^2}{(2a-b)(2a+b)} - \frac{4a^2-b^2}{(2a-b)(2a+b)} =$$

$$\frac{(2a+b)(2a+b)+2(4a^2-4ab-b^2)-2(4a^2-b^2)}{2(2a-b)(2a+b)} =$$

$$\frac{4a^2+4ab+b^2+8a^2-8ab-2b^2-8a^2+2b^2}{2(2a-b)(2a+b)} = \frac{4a^2-4ab+b^2}{2(2a-b)(2a+b)} = \frac{(2a-b)(2a-b)}{2(2a-b)(2a+b)} =$$

$$\underline{\underline{\frac{(2a-b)}{2(2a+b)}}}$$

$$\begin{aligned}
22. \quad & \frac{x-2}{3x-2} - \frac{x+3}{6x+4} - \frac{3x^2-12}{18x^2-8} = \frac{x-2}{(3x-2)} - \frac{x+3}{2(3x+2)} - \frac{3x^2-12}{2(9x^2-4)} = \frac{x-2}{(3x-2)} - \\
& \frac{x+3}{2(3x+2)} - \frac{3x^2-12}{2(3x+2)(3x-2)} = \frac{2(x-2)(3x+2) - (x+3)(3x-2) - (3x^2-12)}{2(3x+2)(3x-2)} = \\
& \frac{2(3x^2+2x-6x-4) - (3x^2-2x+9x-6) - 3x^2+12}{2(3x+2)(3x-2)} = \\
& \frac{6x^2+4x-12x-8-3x^2+2x-9x+6-3x^2+12}{2(3x+2)(3x-2)} = \frac{-15x+10}{2(3x+2)(3x-2)} = \frac{-5(3x-2)}{2(3x+2)(3x-2)} = \\
& \frac{-5}{2(3x+2)}
\end{aligned}$$

$$\begin{aligned}
23. \quad & \frac{1}{c+1} + \frac{6}{c^2-c-2} - \frac{2}{c^2-1} = \frac{1}{(c+1)} + \frac{6}{(c+1)(c-2)} - \frac{2}{(c+1)(c-1)} = \\
& \frac{(c-1)(c-2)+6(c-1)-2(c-2)}{(c+1)(c-1)(c-2)} = \frac{c^2-c-2c+2+6c-6-2c+4}{(c+1)(c-1)(c-2)} = \frac{c^2+c}{(c+1)(c-1)(c-2)} = \\
& \frac{c(c+1)}{(c+1)(c-1)(c-2)} = \frac{c}{(c-1)(c-2)}
\end{aligned}$$

$$\begin{aligned}
24. \quad & \frac{a+1}{2a-1} - \frac{2a}{2a+1} + \frac{7a-2a^2}{1-4a^2} = \frac{a+1}{(2a-1)} - \frac{2a}{(2a+1)} + \frac{7a-2a^2}{-1(4a^2-1)} = \frac{a+1}{(2a-1)} - \\
& \frac{2a}{(2a+1)} + \frac{7a-2a^2}{-1(2a+1)(2a-1)} = \frac{(a+1)(2a+1) - 2a(2a-1) - 1(7a-2a^2)}{(2a+1)(2a-1)} = \\
& \frac{2a^2+2a+a+1-4a^2+2a-7a+2a^2}{(2a+1)(2a-1)} = \frac{-2a+1}{(2a+1)(2a-1)} = \frac{-1(2a-1)}{(2a+1)(2a-1)} = \frac{-1}{(2a+1)}
\end{aligned}$$