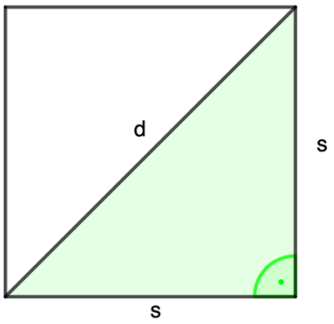
2. Bez Pythagoras Lösungen AB 2

1. Berechne jeweils die Länge der Diagonalen d oder die Seite s in einem Quadrat:



a) $d = \sqrt{7{,}5^2 + 7{,}5^2} = \underline{\underline{10{,}6cm}}$

b) $d = \sqrt{50^2 + 50^2} = \underline{\underline{70{,}7m}}$

c) $d^2 = s^2 + s^2$
$100 = 2s^2 \quad /:2$
$50 = s^2 \quad \sqrt{\ }$
$s = \sqrt{50} = \underline{\underline{7{,}07cm}}$

d) $d^2 = s^2 + s^2$
$2500 = 2s^2 \quad /:2$
$1250 = s^2 \quad \sqrt{\ }$
$s = \sqrt{1250} = \underline{\underline{35{,}4m}}$

e) $d = \sqrt{s^2 + s^2} = \sqrt{2s^2} = \sqrt{2 \cdot 4a^2} = \underline{\underline{2a\sqrt{2}}}$

f) $d^2 = s^2 + s^2$
$64x^2 = 2s^2 \quad /:2$
$32x^2 = s^2 \quad \sqrt{\ }$
$s = \sqrt{32x^2} = \sqrt{16x^2 \cdot 2} = \underline{\underline{4x\sqrt{2}}}$

g) $d = \sqrt{2s^2} = \sqrt{2 \cdot 5\sqrt{3} \cdot 5\sqrt{3}} = \sqrt{6 \cdot 25} = \underline{\underline{5\sqrt{6}}}$

h) $d^2 = s^2 + s^2$
$200 = 2s^2 \quad /:2$
$100 = s^2 \quad \sqrt{\ }$
$s = \underline{\underline{10}}$

2. Berechne an gleichseitigen Dreiecken die Länge der Höhe h oder die Seiten s:

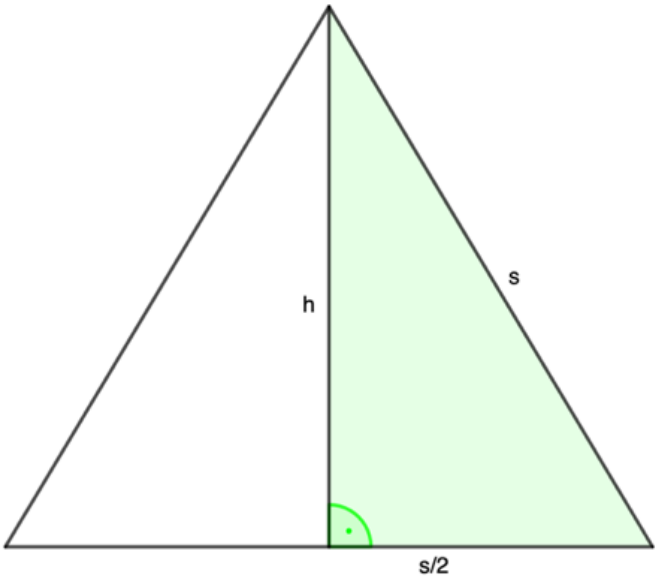
$h^2 = s^2 - \left(\frac{s}{2}\right)^2$

$h^2 = s^2 - \frac{s^2}{4}$

$h^2 = \frac{4s^2}{4} - \frac{s^2}{4}$

$h^2 = \frac{3s^2}{4}$ $\quad /\sqrt{}$

$\underline{h = \frac{s\sqrt{3}}{2}}$



$h^2 = \frac{3s^2}{4}$ $\quad /\cdot 4$

$4h^2 = 3s^2$ $\quad /:3$

$\frac{4h^2}{3} = s^2$ $\quad /\sqrt{}$

$\underline{\frac{2h}{\sqrt{3}} = s}$

a) $h = \sqrt{s^2 - \left(\frac{s}{2}\right)^2} = \sqrt{156{,}25 - 39{,}0625} = \underline{\underline{10{,}8cm}}$

oder $\quad h = \frac{s\sqrt{3}}{2} = \frac{12{,}5 \cdot \sqrt{3}}{2} = \underline{\underline{10{,}8cm}}$

b) $h = \sqrt{s^2 - \left(\frac{s}{2}\right)^2} = \sqrt{400 - 100} = \underline{\underline{17{,}3cm}}$

oder $\quad h = \frac{s\sqrt{3}}{2} = \frac{20 \cdot \sqrt{3}}{2} = \underline{\underline{17{,}3cm}}$

c) $h^2 = s^2 - \left(\frac{s}{2}\right)^2$

$h^2 = \frac{4s^2}{4} - \frac{s^2}{4}$

$9^2 = \frac{3s^2}{4}$ $\quad /\cdot 4$

$324 = 3s^2$ $\quad /:3$

$108 = s^2$ $\quad /\sqrt{}$

$s = \underline{\underline{10{,}4cm}}$ $\qquad$ oder $\quad s = \frac{2h}{\sqrt{3}} = \frac{2 \cdot 9}{\sqrt{3}} = \underline{\underline{10{,}4cm}}$

d) $h^2 = s^2 - \left(\frac{s}{2}\right)^2$

$h^2 = \frac{4s^2}{4} - \frac{s^2}{4}$

$20^2 = \frac{3s^2}{4}$ $/\cdot 4$

$1600 = 3s^2$ $/:3$

$533,\overline{3} = s^2$ $/\sqrt{}$

$s = \underline{\underline{23{,}1m}}$ oder $s = \frac{2h}{\sqrt{3}} = \frac{2 \cdot 20}{\sqrt{3}} = \underline{\underline{23{,}1m}}$

e) $h = \sqrt{s^2 - \left(\frac{s}{2}\right)^2} = \sqrt{64a^2 - 16a^2} = \sqrt{48a^2} = \sqrt{16a^2 \cdot 3} = \underline{\underline{4a\sqrt{3}}}$

oder $h = \frac{s\sqrt{3}}{2} = \frac{8a\sqrt{3}}{2} = \underline{\underline{4a\sqrt{3}}}$

f) $h^2 = s^2 - \left(\frac{s}{2}\right)^2$

$h^2 = \frac{4s^2}{4} - \frac{s^2}{4}$

$(18x)^2 = \frac{3s^2}{4}$ $/\cdot 4$

$1296x^2 = 3s^2$ $/:3$

$432x^2 = s^2$ $/\sqrt{}$

$s = \sqrt{432x^2} = \sqrt{144x^2 \cdot 3} = \underline{\underline{12x\sqrt{3}}}$

oder $s = \frac{2h}{\sqrt{3}} = \frac{2 \cdot 18x}{\sqrt{3}} = \frac{36x \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \underline{\underline{12x\sqrt{3}}}$

g) $h = \sqrt{s^2 - \left(\frac{s}{2}\right)^2} = \sqrt{48 - 12} = \sqrt{36} = \underline{\underline{6}}$ oder $h = \frac{s\sqrt{3}}{2} = \frac{4\sqrt{3} \cdot \sqrt{3}}{2} = \underline{\underline{6}}$

h) $h^2 = s^2 - \left(\frac{s}{2}\right)^2$

$h^2 = \frac{4s^2}{4} - \frac{s^2}{4}$

$\left(27\sqrt{2}\right)^2 = \frac{3s^2}{4}$ $/\cdot 4$

$5832 = 3s^2$ $/:3$

$1944 = s^2$ $/\sqrt{}$

$s = \sqrt{1944} = \sqrt{324 \cdot 6} = \underline{\underline{18\sqrt{6}}}$ oder $s = \frac{2h}{\sqrt{3}} = \frac{2 \cdot 27\sqrt{2}}{\sqrt{3}} = \frac{2 \cdot 27\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \underline{\underline{18\sqrt{6}}}$

3. Kniffligere Aufgabe:

a) Berechne den Flächeninhalt eines gleichseitigen Dreiecks mit der Seite a = 5cm.

$$h = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \sqrt{25 - 6{,}25} = \underline{4{,}33 \ldots cm}$$

$$A = \frac{a \cdot h}{2} = \frac{5 \cdot 4{,}33}{2} = \underline{\underline{10{,}8cm^2}}$$

b) Wie gross ist der Flächeninhalt eines regelmässigen Sechsecks mit einer Seite a = 4cm?

Ein regelmässiges Sechseck besteht aus 6 gleichseitigen Dreiecken.

$$h_{Dreieck} = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \sqrt{16 - 4} = \underline{3{,}464 \ldots cm}$$

$$A_{Dreieck} = \frac{a \cdot h}{2} = \frac{4 \cdot 3{,}464}{2} = \underline{6{,}928 \ldots cm^2}$$

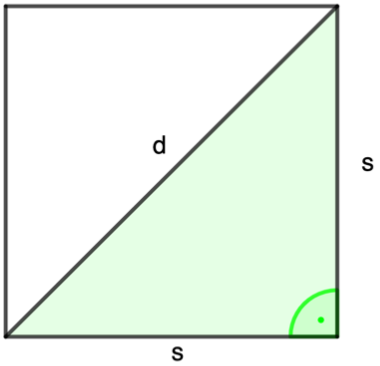
$$A_{Sechseck} = 6 \cdot A_{Dreieck} = \underline{\underline{41{,}6cm^2}}$$

4. Berechne Seite und Flächeninhalt eines Quadrates aus seiner Diagonalen:

$d^2 = s^2 + s^2 = 2s^2$ /:2

$\frac{d^2}{2} = s^2$ /√

$\underline{s = \frac{d}{\sqrt{2}}}$



a) $d^2 = s^2 + s^2$

$36 = 2s^2$ /: 2

$18 = s^2$ √

$s = \sqrt{18} = \underline{\underline{4{,}2cm}}$ oder $s = \frac{d}{\sqrt{2}} = \frac{6}{\sqrt{2}} = \underline{\underline{4{,}2cm}}$

$A = s^2 = \underline{\underline{18cm^2}}$

b) $d^2 = s^2 + s^2$

$5 = 2s^2 \quad /:2$

$2{,}5 = s^2 \quad \sqrt{}$

$s = \sqrt{2{,}5} = \underline{\underline{1{,}6cm}}$ oder $s = \frac{d}{\sqrt{2}} = \frac{\sqrt{5}}{\sqrt{2}} = \underline{\underline{1{,}6cm}}$

$A = s^2 = \underline{\underline{2{,}5cm^2}}$

c) $d^2 = s^2 + s^2$

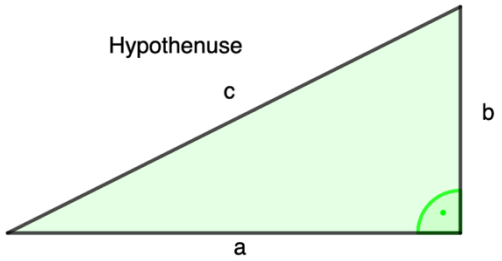
$0{,}\overline{4} = 2s^2 \quad /:2$

$0{,}\overline{2} = s^2 \quad \sqrt{}$

$s = \sqrt{0{,}\overline{2}} = \underline{\underline{0{,}47m}}$ oder $s = \frac{d}{\sqrt{2}} = \frac{\frac{2}{3}}{\sqrt{2}} = \underline{\underline{0{,}47m}}$

$A = s^2 = \underline{\underline{0{,}22m^2}} = \underline{\underline{\frac{2}{9}m^2}}$

5. Berechne die Hypotenuse (längste Seite) eines rechtwinkligen Dreiecks mit der einen Kathete a und der anderen Kathete b.



a) $c = \sqrt{a^2 + (2a)^2} = \sqrt{a^2 + 4a^2} = \sqrt{5a^2} = \underline{\underline{a\sqrt{5}}}$

b) $c = \sqrt{a^2 + (3a)^2} = \sqrt{a^2 + 9a^2} = \sqrt{10a^2} = \underline{\underline{a\sqrt{10}}}$

c) $c = \sqrt{a^2 + \left(\frac{a}{2}\right)^2} = \sqrt{a^2 + \frac{a^2}{4}} = \sqrt{\frac{4a^2}{4} + \frac{a^2}{4}} = \sqrt{\frac{5a^2}{4}} = \underline{\underline{\frac{a\sqrt{5}}{2}}}$

d) $c = \sqrt{a^2 + \left(\frac{2a}{3}\right)^2} = \sqrt{a^2 + \frac{4a^2}{9}} = \sqrt{\frac{9a^2}{9} + \frac{4a^2}{9}} = \sqrt{\frac{13a^2}{9}} = \underline{\underline{\frac{a\sqrt{13}}{3}}}$

e) $c = \sqrt{a^2 + \left(a\sqrt{3}\right)^2} = \sqrt{a^2 + 3a^2} = \sqrt{4a^2} = \underline{\underline{2a}}$